# ASSESSING SPATIAL VARIATION OF CORN RESPONSE TO IRRIGATION USING A BAYESIAN SEMIPARAMETRIC MODEL

K. C. Stone, E. J. Sadler

***A Tribute to the Career of Terry Howell, Sr.***

**ABSTRACT.** *Spatial irrigation of agricultural crops using site-specific variable-rate irrigation (VRI) systems is beginning to have widespread acceptance. However, optimizing the management of these VRI systems to conserve natural resources and increase profitability requires an understanding of the spatial crop responses. In this research, we utilize a recently developed spatially explicit analysis model to reanalyze spatial corn yield data. The specific objectives of this research were to (1) calculate a suite of estimates (estimated yield, rainfed yield, maximum yield, and irrigation at maximum yield) and provide credible intervals (measures of uncertainty) around these estimates for comparing with the previous analysis, and (2) examine whether the conclusions from this rigorous re-analysis were different from the prior analysis and if the results would force any modifications to the conclusions obtained with the prior analyses. The spatially explicit analysis was achieved using a mixed model formulation of bivariate penalized smoothing splines and was implemented in a Bayesian framework. This model simultaneously accounted for spatial correlation as well as relationships within the treatments and had the ability to contribute information to nearby neighbors. The model-based yield estimates were in excellent agreement with the observed spatial corn yields and were able to estimate the high and low yields more accurately than the previous analysis. Credible intervals were calculated around the estimates, and the majority encompassed the observed yields. After calculating estimates of yield, we then calculated estimates of other response variables, such as rainfed yield, maximum yield, and irrigation at maximum yield. These estimated response variables were then compared with previous results from a classical statistical analysis. Our conclusions supported the original analysis in identifying significant spatial differences in crop responses across and within soil map units. These spatial differences were great enough to be considered in irrigation system design and management. The major improvement in the 2014 re-analysis is that the model explicitly considered spatial dependence in calculating the estimated yields and other variables and thus should provide improved estimates of the impact of spatial differences for use in irrigation system design and management.*

**Keywords.** *Additive models, Bivariate smoothing, Penalized splines, Response curves, Semiparametric regression, Site-specific agriculture, Varying coefficient models.*

Irrigation is recognized as the worldwide largest use of freshwater, at ~70% of freshwater withdrawals (Kassam et al., 2007; Molden et al., 2007). Generally speaking, freshwater is a finite resource, so rising regional populations create disturbing trends in per capita freshwater use in many regions (Molden et al., 2007). Significant populations in several regions of the world are subsisting on less than adequate daily caloric intake, and in some regions the per capita trends in irrigation water resources are not promising for providing additional food.

In addition, water has seen markedly increased contention for municipal, industrial, transport/navigation, recreational, and ecological uses in recent decades. Economic and regulatory forces have prevailed and in some cases nearly eliminated nearby irrigated agriculture (Boehlert and Jaeger, 2010). In many locations previously unaffected by water contention, litigation has been used to alter water use or at least put added pressure on the economic value of water to the point that water rights have been transferred from agriculture, usually to municipalities (Brown, 2003).

Thus, extensive attention has been directed toward increasing food production per unit of water used, either as withdrawals in irrigated production or per unit of evapotranspiration in rainfed production. Molden (2007) summarized a spectrum of approaches that show potential for improving the so-called "crop per drop." These include crop genetics to improve yield or increase drought tolerance, irrigation management to reduce water use while maintaining yields, improved water conveyance methods to reduce evaporation and other losses, and methods to reduce soil evaporation and

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The authors are **Kenneth C. Stone, ASABE Member,** Agricultural Engineer, USDA-ARS Coastal Plains Soil, Water, and Plant Research Center, Florence, South Carolina; **E. John Sadler, ASABE Member,** Soil Scientist, USDA-ARS Cropping Systems and Water Quality Research Unit, Columbia, Missouri. **Corresponding author:** Kenneth C. Stone, USDA-ARS Coastal Plains Soil, Water, and Plant Research Center, 2611 West Lucas St., Florence, SC 29501-1242; phone: 843-669-5203; e-mail: ken.stone@ars.usda.gov.

thus shift water consumption from evaporation to transpiration. Of these, operational irrigation management (e.g., more precise scheduling, deficit irrigation, timely initiation, and termination on a seasonal basis) can perhaps be improved more rapidly and with less capital investment than the other approaches. At the next increment of investment, there seems to be substantial potential for savings with modest cost in irrigation infrastructure. One example is plastic tubing to simultaneously fill rice paddies rather than letting the water cascade from upper to lower paddies in the field, which has been shown to save ~24% of irrigation water compared with conventional practice (Vories et al., 2005).

With the convergence of geopositioning (GPS), geospatial information (GIS), and increasingly powerful computing technologies in the 1980s and 1990s, much attention has been given to precision, or site-specific, agriculture as a way to improve the economics and environmental benefits of agriculture. Many crop inputs, including fertilizer, plant population, and pest control, have been examined for their potential to reduce costs or increase yield, or perhaps shift the input from where it is not used efficiently to other places in the field where it is. Considering irrigation as an input to be managed concurrently with all other inputs, a logical extension is to consider if spatially varying irrigation could be a water-conserving approach or at least an approach for gaining more yield per unit of water used. This was demonstrated to be the case based on research in South Carolina (Sadler et al., 2005), where optimal variable-rate irrigation had potential to save 10% to 15% in three fairly dry years (1999-2001). It was also shown in two cases in Georgia, where from slightly more to 36% less water was used in 2003 with variable-rate irrigation systems (http://water.usgs.gov/wrri/02-03grants_new/prog-compl-reports/2003GA38B.pdf). The results were sufficiently promising that the USDA-NRCS awarded a three-year demonstration grant to retrofit six irrigation systems per year in Georgia and South Carolina (www.nrcs.usda.gov/Internet/FSE_DOCUMENTS/stelprdb 1045308.pdf). Clearly, the water-conserving aspects of variable rate irrigation merit further examination.

## Specific Case of Site-Specific Irrigation

One data set that has potential to inform this question was obtained in Florence, South Carolina, at a variable-rate center-pivot research facility (Camp and Sadler, 1998; Camp et al., 1998, Sadler et al., 2002). In 1996 and 1998, two commercial systems were retrofitted with a research-grade variable-rate irrigation system (Omary et al., 1997). The system had the ability to independently irrigate areas as small as $9.1 \times 9.1$ m, with the central $6.1 \times 6.1$ m area considered uniformly irrigated. System tests confirmed that the performance was within expectations (Camp et al., 1998; Stone et al., 2006). A three-year corn yield experiment was conducted to examine the simultaneous effects of irrigation and N fertilization. The experiment was intended to be analyzed by both analysis of variance (ANOVA) and geostatistical analyses. The constraints on the experimental design were discussed in detail by Sadler et al. (2002). In brief, four irrigation treatments, i.e., 0%, 50%, 100%, and 150% of irrigation to replace evapotranspiration (ET), were applied simultaneously in all irrigated plots when the mean soil water tension in the 100% plots in four (1999) or six (2000-2001) soil map units was less than or equal to -30 kPa at the 0.3 m depth. There were two N treatments: 134 kg ha$^{-1}$, which is the extension recommendation for rainfed corn in South Carolina, and 225 kg ha$^{-1}$ for irrigated corn. Nitrogen fertilizer was a blended dry granular preplant application common to all plots within a year, followed by treatments applied as urea-ammonium nitrate with sulfur (UAN 24S) injected in a nominal 13, 11, and 16 mm irrigation in all plots during the three years. Where sufficient area existed within a map unit, one or more randomized complete blocks (RCB) of these eight treatments were imposed. Where insufficient area existed, a randomized incomplete block (RICB) was used. In total, there were 39 RCBs and 19 RICBs, for a total of 396 plots. Harvest was done with a plot combine from two rows 6.1 m in length near the center of the plots. Additional details are given by Sadler et al. (2002).

### *Previous Analysis of Variance*

The ANOVA results and specific models used to analyze the previous experiment were also described by Sadler et al. (2002). In summary, the irrigation main effect on yield was, as expected, highly significant in both linear and quadratic contrasts over the applied range of 0% to 150% of full irrigation. Importantly, the deviation from quadratic contrast was not significant in any year. Somewhat surprisingly, however, between the two N fertilizer rates of 134 and 225 kg ha$^{-1}$, the main effect of N was significant in only one of three years. The test of whether the soil variation was a significant contributor to variation in yield was significant, despite the limitations of the experimental design. Interaction effects were not consistent, either in two-way or three-way interactions. In short, the soil variation was important, the expected irrigation effect was obtained, and the N effect was somewhat surprising. Sadler et al. (2002) further evaluated the quadratic irrigation production functions for map unit means in that experiment and solved for maximum yield and the irrigation required to obtain it. These analyses were all done with soil map units as a class variable and did not explicitly account for spatial variation within blocks, within soil map units, or among soil map units. The only indication of spatial variation in the production functions was that block-level functions for the most common soil map unit were provided. Further, the map-unit-mean analytical expressions for yield as a function of irrigation were obtained separately for N treatments in only 2001, the year in which the N treatment was significant. These characteristics of the analyses limited interpretation of the spatial variation in the production functions themselves.

Results from this and a nearby experiment under the other site-specific center pivot were subjected to marginal benefit analyses to determine the economically optimum irrigation for corn (Lu et al., 2004a) and the simultaneous optimum irrigation and N fertilization for corn (Lu et al., 2004b). The data described previously were also used for a study examining the economic feasibility of spatially variable irrigation for the Southeast Coastal Plain (Lu et al., 2005). The primary analytical tool was multiple regression to obtain yield response surfaces as a function of irrigation and N fertilization. These analyses were conducted under the 1999-2001 corn and irrigation price estimates. In the period since those studies, both these values have changed significantly.

***Two-Stage Spatial Regression Analysis***

A first attempt to explicitly account for spatial variation in these data was described by Sadler et al. (2002). The goal of this analysis was not to test statistical significance but rather to allow generation of maps of derived characteristics, including rainfed yield, maximum yield, and irrigation water use efficiency, in addition to maps of the irrigation required to achieve those characteristics. (One by-product of the analytical examination was the realization that the slope of the production function evaluated at zero irrigation would be the marginal benefit of the first increment of water applied. This could be interpretable as the marginal benefit of water-conserving technologies in rainfed regimes.) All derived results were obtained from averages over N treatment, on the basis that the N treatment was not significant in two of three years. This analysis involved two steps: a separate interpolation of yields from each individual treatment to estimate the yield that would have been expected for all four irrigation treatments at each of the 396 plot centers, and a quadratic regression of the four yields on seasonal irrigation in mm.

Sadler et al. (2003) described the marginal N response for given irrigation levels and explained the two-step process for all eight treatments, producing analytical expressions of irrigation production functions for both N treatments at each of the 396 plot centers. The maps of derived surfaces qualitatively showed distinct spatial patterns in the field. The primary conclusion was that spatial patterns in marginal N response were not stable across years and further that marginal N response was surprisingly variable, ranging from negative to positive in credible areas (i.e., multiple data points in each area) of the field. These results demonstrated the utility of having mathematical expressions for irrigation production functions at many areas within a field and were, at that time, to our knowledge, the only such results in existence. Using spatial patterns of maximum yield, rainfed yield, irrigation water use efficiency, and prices, the marginal economic benefits of irrigation and N can be provided. However, the procedure suffers a number of limitations, including dependence on only spatial variation for step one and on only irrigation for step two (N is accounted for by performing regressions separately for the two N treatments). There is also no good means to provide estimates of uncertainty (variation) around the yield estimates themselves, nor to determine significance in the estimated yield.

***Spatially Explicit Analysis***

Despite the benefits of the analyses previously performed, these data required a spatially explicit analysis. The spatially explicit analysis was achieved using a Bayesian mixed model formulation of bivariate penalized smoothing splines (Holan et al., 2008). This model simultaneously accounted for spatial correlation as well as relationships within the treatments; in other words, it considered $X$, $Y$, irrigation, and N, all with the ability to contribute information to nearby neighbors. The outcomes of this analysis were required to include yield estimates, analytical expressions for irrigation and N production functions, and estimates of the uncertainty in yield estimates, coefficients of the analytical expressions, and in the derived variables (e.g., maximum, rainfed, and economic yield), the irrigation required to provide them and irrigation water use efficiency or water productivity. Additionally, the model provided credible intervals around the estimated variables. This article employs the method of Holan et al. (2008) to re-analyze the experimental data of Sadler et al. (2002) using a spatially explicit analysis. The specific objectives of this research were to (1) calculate values for estimated yield, rainfed yield, maximum yield, and irrigation at maximum yield and provide credible intervals around those estimated variables for comparing with the previous analysis, and (2) examine whether the results of this rigorous reanalysis differed from the prior analysis and whether the results force any modifications to the conclusions obtained with the prior analyses.

# METHODS

The observed spatial corn yield data (as described above in the Specific Case of Site-Specific Irrigation section) collected from Sadler et al. (2002) (2002 analysis) were used to estimate values for a suite of variables (estimated yield, rainfed yield, maximum yield, and irrigation at maximum yield) using the recently developed spatially explicit analysis model (2014 analysis). The 2014 analysis, which used the method of Holan et al. (2008), input the spatial coordinates, imposed irrigation and nitrogen (N) treatments, and observed yield data to estimate the spatial yields. Additionally, the 2014 analysis provided credible intervals (posterior distributions) for the individual estimated yields and used a spatially treatment-varying coefficient model to fit the observed yield data.

For a general comparison of the performance of the 2002 and 2014 approaches, the results of both sets of yield estimates were compared to the observed yields using linear regression (SAS Institute, Inc., Cary, N.C.). For a comparison of derived analysis variables (maximum and rainfed yield, and irrigation at maximum yield), standard summary statistics (means, standard deviations, minimum and maximum values) were calculated using SAS, and the differences between the estimated variables were calculated (i.e., 2002 versus 2014). The point estimates from both analyses were also compared using linear regression (i.e., perfect agreement would result in intercept = 0, slope = 1, and $r^2 = 1.0$). Contour maps of the calculated variables were generated using Surfer software (Golden Software, Inc., Golden, Colo.) using default interpolation parameters (point kriging, slope = 1, aniso = 1.0).

# RESULTS

## APPLYING THE SPATIALLY EXPLICIT MODEL FOR EXPERIMENTAL DATA ANALYSIS

The first question to be answered for this or any interpolation method is whether or not the specified model provides a good fit of the data. An initial evaluation can usually be derived from regressions of model estimates to the observed value ($r^2$, RMSE, intercept, slope, etc.) and analysis of the residuals. A more rigorous evaluation might involve bootstrap methodology or other more sophisticated methods that attempt to account for the uncertainty in the estimates from the original two-stage approach (frequentist procedure). In principle, such procedures would be possible but would be more computationally intensive, and thus they are not pursued here.

We present the interpolation results of the 2014 analysis and compare them with the observed values and the results of the 2002 analysis. This comparison allows us to proceed with objectives 1 and 2 of this article while providing an assessment of the potential limitations of the current approach.

### *Estimated Yield*

The observed yields were fit using the spatially explicit analysis (Holan et al., 2008). The 1999-2001 estimated corn yields were calculated using the 2014 analysis (figs. 1 through 3). The yield estimates were then plotted against the observed yields for comparison (figs. 4 through 6). In 1999, the 2014 analysis estimated the yields very well in terms of $r^2$. The estimated yield slope was 0.82 Mg mm$^{-1}$ ($r^2$ = 0.83, RMSE = 0.82; fig. 4). The 1999 growing season was generally considered a drought year; it required the greatest irrigation depths and had the largest variation in corn yields of all three years. The 2002 yield estimates were also plotted for comparison (slope = 0.73 Mg ha$^{-1}$, $r^2$ = 0.85, RMSE = 0.68; fig. 5). The 2014 yield estimate had a slope closer to 1.0, indicating that the 2014 analysis did a better job of estimating the larger and smaller yields than the 2002 analysis. Additionally, the upper and lower credible intervals for the 2014 estimated yields were plotted along with the regression (fig. 5). In 2000, the estimated yield slope was 0.78 Mg ha$^{-1}$ and had an $r^2$ of 0.80 (fig. 5). The 2000 slope was less than that in 1999 and may be attributed to the smaller irrigation amounts applied. Again, the 2002 analysis estimated slope was less than that found in the 2014 analysis. The 2001 season had the smallest irrigation applications of the three-year study, indicating better weather than in either of the first two years and correspondingly smaller observed variation in yield. The 2014 estimated yield slope was 0.64 Mg ha$^{-1}$ ($r^2$ = 0.66, RMSE = 0.67; fig. 6), while the 2002 estimated yield slope was much smaller, 0.39 Mg ha$^{-1}$ ($r^2$ = 0.77, RMSE = 0.32). The overall estimated yield fit over the three years decreased from 1999 to 2001 due to the decreasing irrigation depths required and the corresponding decreased variation in corn yields. The 2001 growing season was considered a more typical rainfall year than the two earlier years. In each year, the slopes of the two analyses were significantly different. It appears that the 2014 analysis was able to estimate the corn yields slightly better than the 2002 analysis over the three-year study period. This is a result of the two approaches: the 2002 analysis used blocks of measured yield to calculate yield response curves, whereas the 2014 analysis approach estimated yield response curves using the entire sample population, taking into account the spatial dependence.

## Comparison of 2014 Predicted Variables to 2002 Estimates

In our analysis, there are several quantities of interest that can all be obtained directly as output from the spatially treatment-varying coefficient model or as deterministic transformations of this output. The items of interest in this analysis are rainfed yield (i.e., yield when irrigation is zero), maximum yield, irrigation that produced maximum yield, economic optimum yield, and irrigation that produced the economic optimum yield. Other variables could be estimated for additional analysis; however, due to space constraints, only those identified here will be discussed.

### *Rainfed Yield*

The rainfed yield estimates, particularly in drought years, can provide irrigation designers a good initial estimate of the potential areas of a field where irrigation may provide the most benefit. The 1999 and 2000 corn growing seasons were generally considered drought years. In these two years and for each estimation method, estimated rainfed yields were similar (table 1; figs. 7 and 8). The 1999 mean rainfed yields were 6.75 and 6.44 Mg ha$^{-1}$, respectively, for the 2014 and 2002 estimation methods, and the 2000 mean rainfed yields were 5.7 and 5.3 Mg ha$^{-1}$, respectively. In 1999, the 2014 analysis estimated rainfed yields ranging from 2.6 to 10.4 Mg ha$^{-1}$, and the 2002 analysis estimated rainfed yields ranging from 3.7 to 8.2 Mg ha$^{-1}$. The larger ranges between the minimum and maximum rainfed yields for 1999 for the two estimation methods is due to the 2002 analysis using only points that were in the same irrigation treatment, which reduced the number of points used in estimating the response surface. The 2014 analysis used the entire data set in predicting the estimated rainfed yields and retained the influence of extreme values. In both the 1999 and 2000 contour plots (figs. 5 and 6), there appears to be a consistent area from upper left to lower right that has relatively greater rainfed yields, indicating that this area would be the most productive region of the field under rainfed conditions. The 2001 growing season was a more typical rainfall year, and there were less defined regions within the field (data not shown).

Additionally, we did a simple linear regression comparison of the 2014 and 2002 analysis methods (table 2). The slopes of the regression analysis from 1999 to 2001 were 0.65, 0.32, and 0.41, respectively, indicating that the 2014 analysis preserved the spatial variation better than the 2002 analysis, which averaged out more of the spatial variation.

### *Maximum Yield*

The maximum yield estimates provide the potential spatial yields achievable under ideal conditions. The 2001 maximum calculated yields were larger than the 1999 and 2000 maximum yields (table 1; fig. 9). For the 2002 analysis, the 1999 to 2001 maximum yields ranged from 8.7 to 12.2 Mg ha$^{-1}$, from 8.7 to 11.6 Mg ha$^{-1}$, and from 10.6 to 12.8 Mg ha$^{-1}$, respectively, with mean maximum yields of 10.7, 10.4, and 11.7 Mg ha$^{-1}$, respectively. For the 2014 analysis, the 1999 to 2001 maximum yields ranged from 5.8 to 12.7 Mg ha$^{-1}$, from 6.2 to 13.3 Mg ha$^{-1}$, and from 9.2 to 14.4 Mg ha$^{-1}$, respectively, with mean maximum yields of 10.9, 10.6, and 12.0 Mg ha$^{-1}$, respectively.

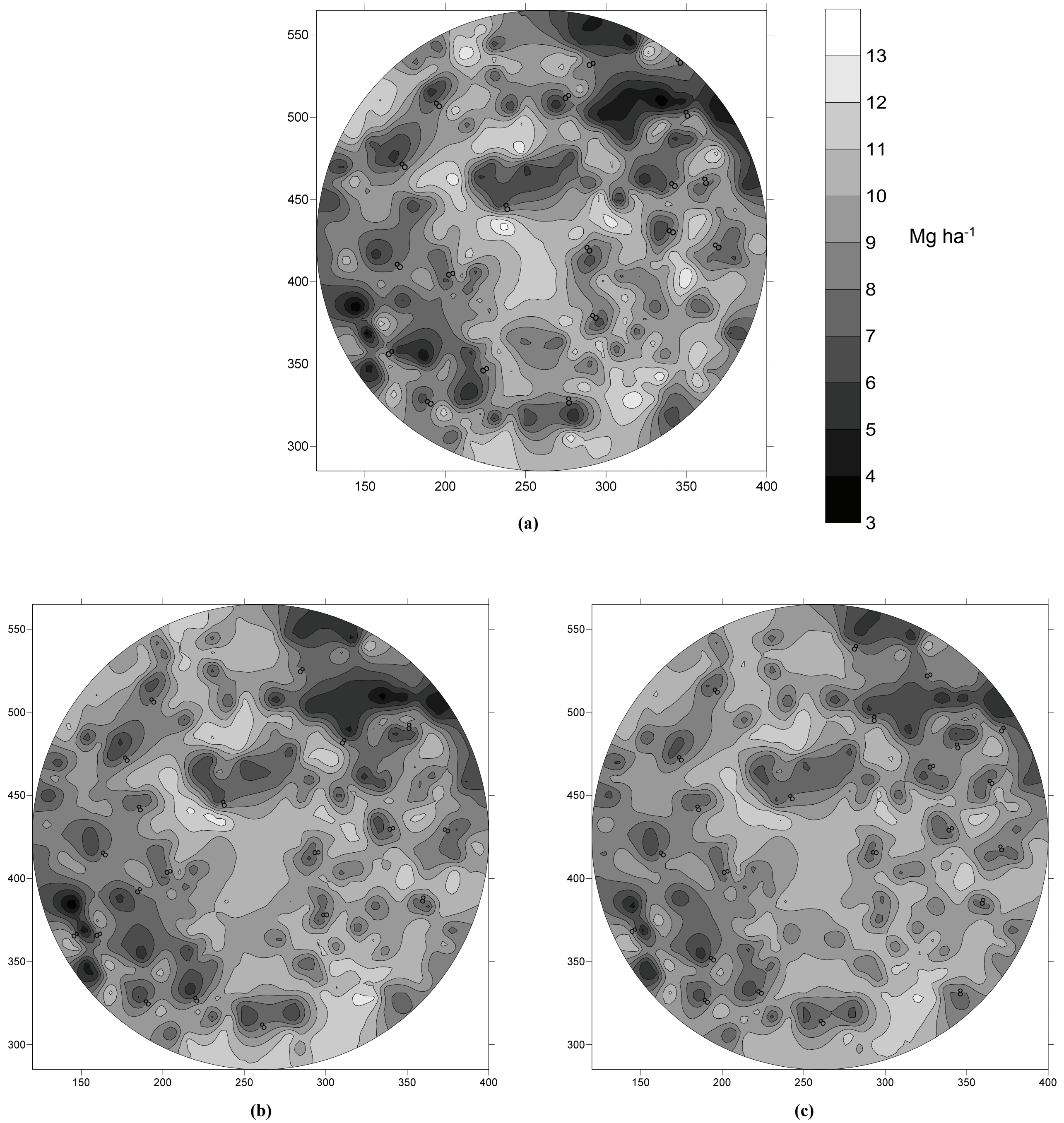



**Figure 1. (a) Observed yield and estimated yield maps for the (b) 2014 analysis and (c) 2002 analysis for 1999.**

For all years, the differences in maximum yields ranged from -4.33 to 2.5 Mg ha$^{-1}$. The maximum yield regression $r^2$ values from 1999 to 2001 were 0.60, 0.54, and 0.39, respectively (table 2). This reduced $r^2$ value over time (1999 to 2001) was similar to that observed for the predicted yields and possibly due to the better weather in 2001.

### *Irrigation at Maximum Yield*

Estimation of the irrigation required for maximum yields can provide irrigation system designers the appropriate parameters for calculating maximum water application rates and design flow rates. The irrigation depth corresponding to the calculated maximum yield using the 2014 analysis ranged from 186 mm in 2001 to 282 mm in 1999 (table 1; fig. 10), compared to the 2002 analysis, which ranged from 204 in 2001 to 286 mm in 1999. The contour maps created for the irrigation depth at maximum irrigation illustrate the differences between the two estimation methods. The calculated response from the 2002 analysis resulted in areas of the field with little detail. The 2014 analysis used the entire dataset and was able to fill areas that were very flat in the 2002 analysis. In comparing the two methods using regression analysis, the slopes were 0.62, 0.39, and 0.23, for 1999, 2000, and 2001, respectively. However, from 1999 to 2001, for irrigation at maximum yield, the regression $r^2$ values were 0.43, 0.19, and 0.05, respectively, indicating poor cor-

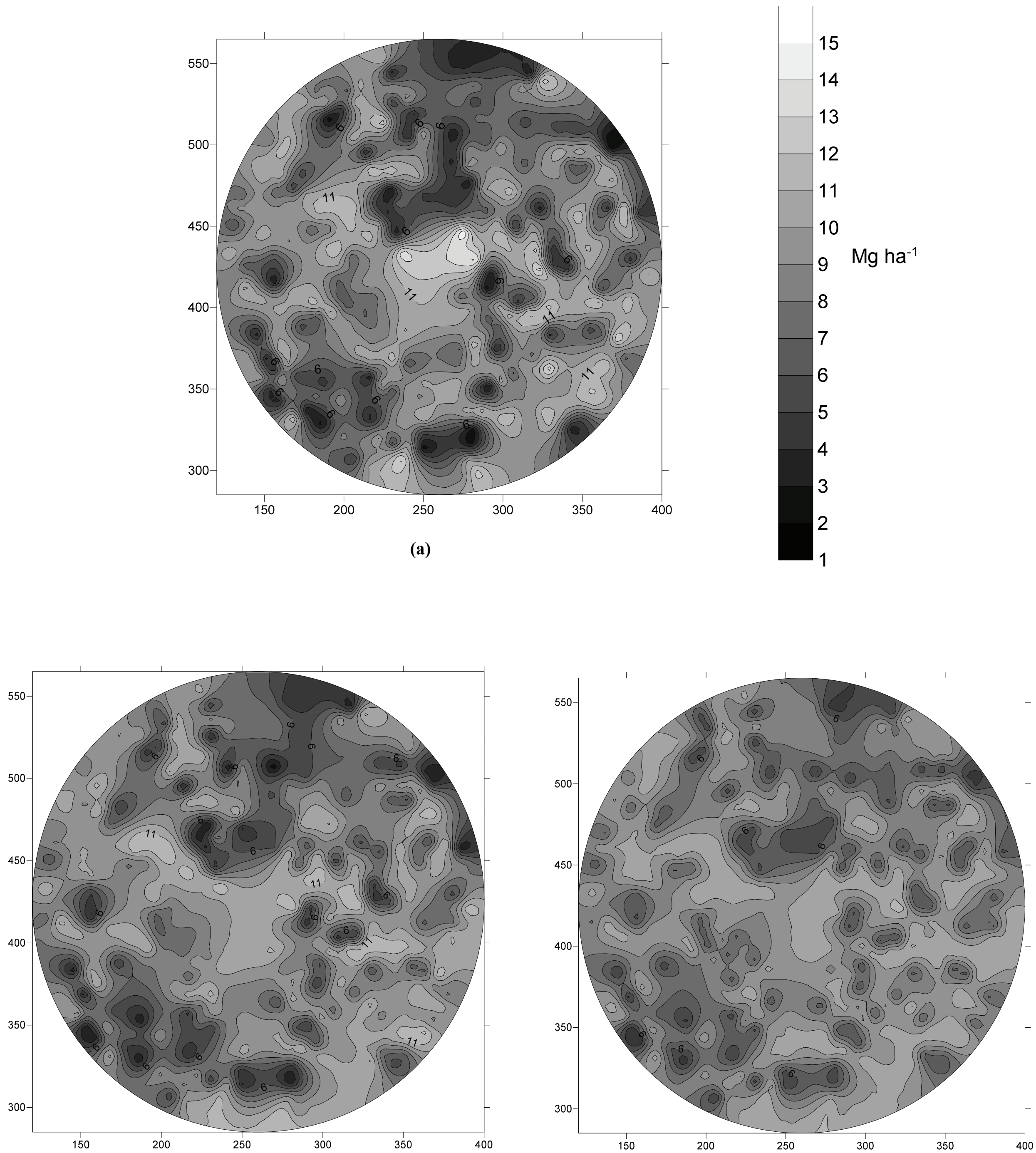



**Figure 2. (a) Observed yield and estimated yield maps for the (b) 2014 analysis and (c) 2002 analysis for 2000.**

relation between the two analyses. Clearly, the results obtained from the 2014 analysis provide more information for irrigation system design on this and other fields with similarly large variation in the soil resource.

# Summary and Conclusions

The recently developed spatially explicit analysis (2014 analysis) was used to re-analyze spatial corn yield data. The estimated yields from the 2014 method were in excellent agreement with the observed spatial corn yields. The 2014 analysis preserved more of the spatial variation in the predicted yields and response variables. Overall, the 2014 analysis predicted mean estimated yields for each response variable in relatively close agreement to the 2002 analyses and also provided uncertainty estimates. The prediction of irrigation at maximum yield provided the poorest correlation between the two methods over the three years.

Our second objective asked if the 2014 analysis would change the conclusion reached in the 2002 analysis. The 2002 analysis concluded that (1) significant differences existed in the response of corn to irrigation, both across soil

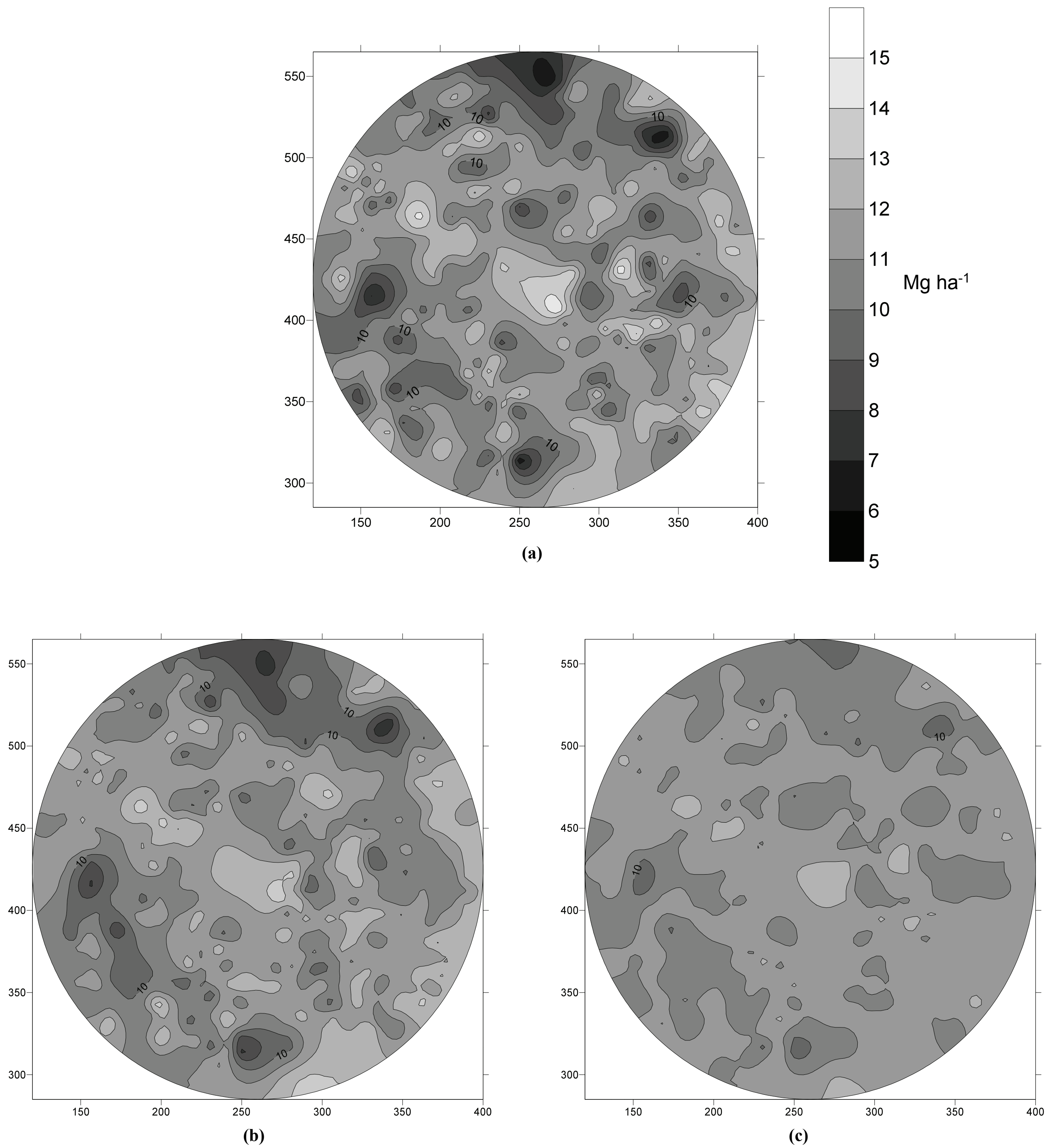



**Figure 3. (a) Observed yield and estimated yield maps for the (b) 2014 analysis and (c) 2002 analysis for 2001.**

map units and within soil map units; (2) differences between soil map units existed at magnitudes that would likely be important in irrigation system design and management; and (3) irrigation system managers and designers should consider the effects of unexpectedly large spatial variation in crop response. Furthermore, in a follow-up analysis, Sadler et al. (2005) concluded that, in addition to corn response to irrigation varying spatially, both rainfed yields and calculated maximum irrigated yields also exhibited spatial patterns.

In our re-analysis of these data, we confirmed the conclusions of Sadler et al. (2005). However, unlike the 2002 analysis, the 2014 analysis specifically accounted for spatial dependence and provided measures of uncertainty, and it is therefore more rigorous and intellectually satisfactory. The model coefficients from the 2014 analysis were spatially varying, resulting in model estimates and credible intervals obtained using all of the observations simultaneously and adding confidence to the results. In all, the 2014 analysis can provide additional insights into the spatial responses of crops to irrigation, and it could be used to provide irrigation managers and designers with tools needed to make critical water management decisions.

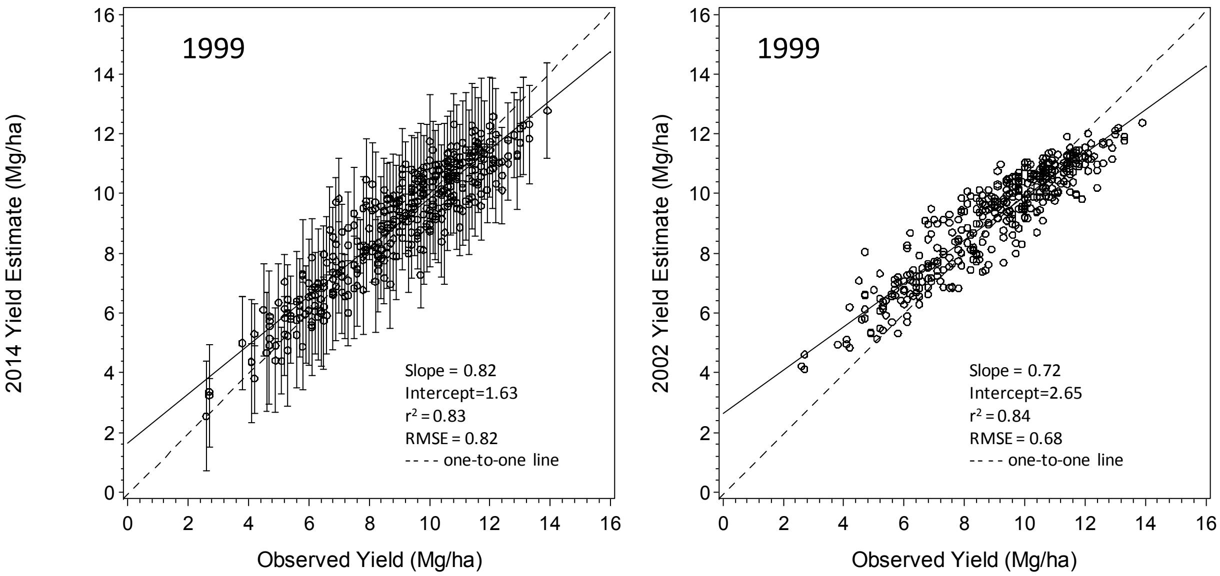



**Figure 4. Regression of the 1999 estimated yields versus observed yields for the 2014 and 2002 analyses. The 2014 analysis provides 95% credible intervals for the estimates.**

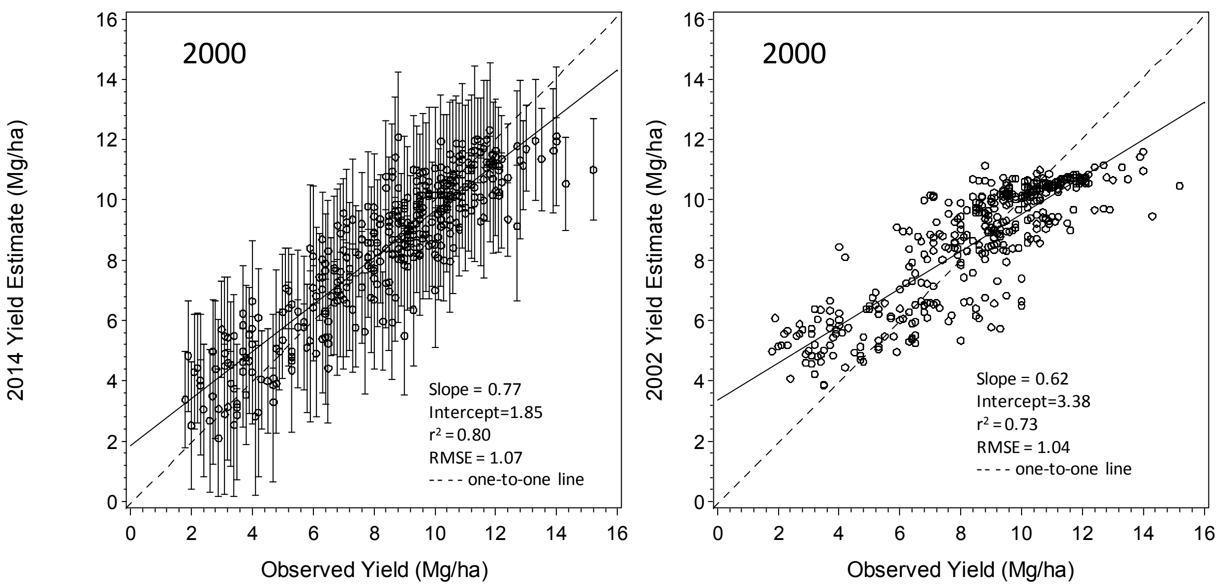



**Figure 5. Regression of the 2000 estimated yields versus observed yields for the 2014 and 2002 analyses. The 2014 analysis provides 95% credible intervals for the estimates.**

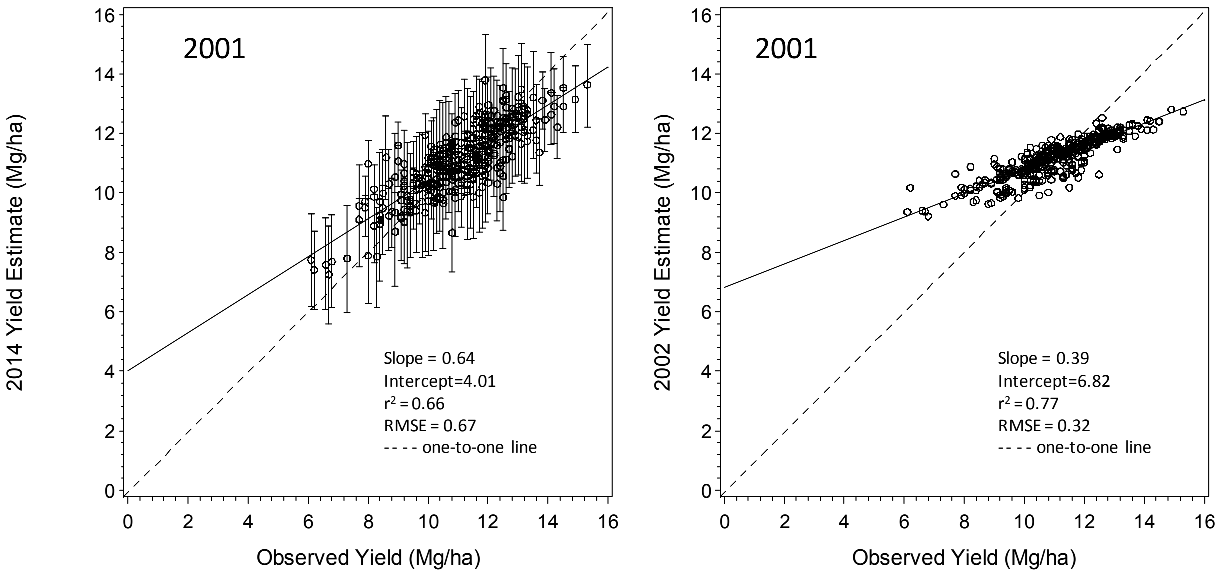
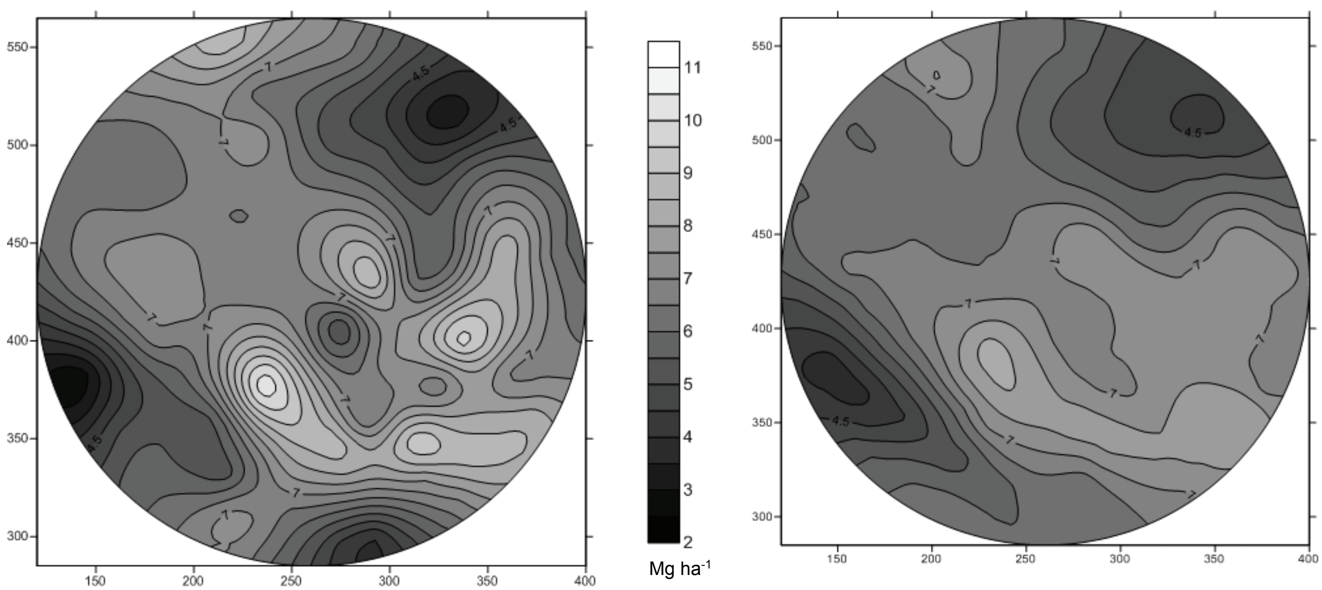



**Figure 6. Regression of the 2001 estimated yields versus observed yields for the 2014 and 2002 analyses. The 2014 analysis provides 95% credible intervals for the estimates.**

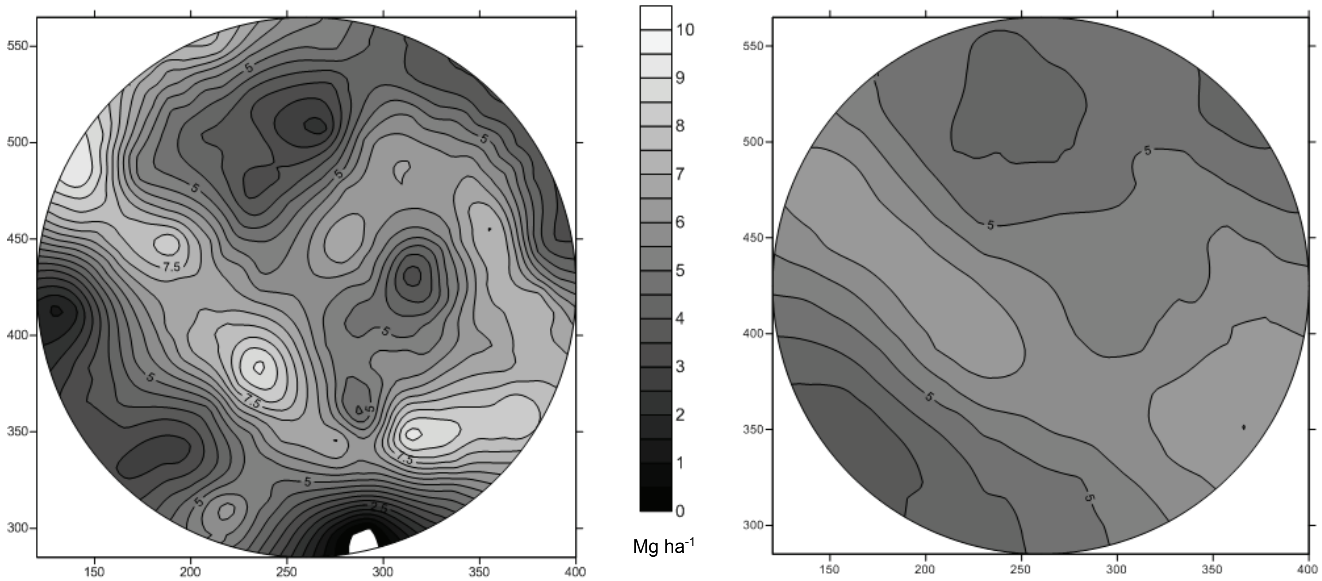
**Table 1. Comparison of summary statistics associated with the Bayesian (2014 analysis) estimated yield, rainfed yield, maximum yield, and irrigation at maximum yield, to those from Sadler et al. (2002) (2002 analysis) on a point-by-point basis.**

| | 1999 | | | | 2000 | | | | 2001 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Variable | Mean | SD | Min. | Max. | Mean | SD | Min. | Max. | Mean | SD | Min. | Max. |
| Observed yield (Mg ha$^{-1}$) | 9.06 | 2.20 | 2.60 | 13.9 | 8.49 | 2.73 | 1.80 | 15.20 | 11.10 | 1.48 | 6.10 | 15.30 |
| Yield estimate (Mg ha$^{-1}$) | | | | | | | | | | | | |
| 2014 analysis | 9.06 | 1.98 | 2.53 | 12.77 | 8.47 | 2.38 | 2.09 | 12.31 | 11.10 | 1.16 | 7.25 | 13.78 |
| 2002 analysis | 9.22 | 1.74 | 4.10 | 12.36 | 8.61 | 1.97 | 3.84 | 11.60 | 11.20 | 0.66 | 9.21 | 12.79 |
| Difference | -0.165 | 0.463 | -2.493 | 1.583 | -0.143 | 0.866 | -2.815 | 2.190 | -0.093 | 0.690 | -2.766 | 2.196 |
| Rainfed yield (Mg ha$^{-1}$) | | | | | | | | | | | | |
| 2014 analysis | 6.75 | 1.33 | 2.62 | 10.43 | 5.66 | 1.63 | -0.05 | 9.49 | 10.34 | 0.96 | 6.97 | 12.59 |
| 2002 analysis | 6.44 | 0.96 | 3.70 | 8.20 | 5.27 | 0.69 | 3.60 | 6.50 | 10.25 | 0.53 | 8.80 | 11.50 |
| Difference | 0.315 | 0.628 | -2.078 | 2.326 | 0.393 | 1.196 | -5.049 | 3.347 | 0.085 | 0.670 | -2.574 | 1.733 |
| Maximum yield (Mg ha$^{-1}$) | | | | | | | | | | | | |
| 2014 analysis | 10.86 | 1.00 | 5.79 | 12.71 | 10.57 | 1.22 | 6.21 | 13.31 | 11.99 | 0.92 | 9.24 | 14.42 |
| 2002 analysis | 10.79 | 0.56 | 8.70 | 12.20 | 10.43 | 0.40 | 8.70 | 11.60 | 11.69 | 0.33 | 10.60 | 12.80 |
| Difference | 0.063 | 0.669 | -4.332 | 1.836 | 0.146 | 0.964 | -3.192 | 2.174 | 0.298 | 0.758 | -2.260 | 2.515 |
| Irrigation at maximum yield (mm) | | | | | | | | | | | | |
| 2014 analysis | 282.05 | 30.86 | 161.48 | 320.05 | 249.82 | 25.67 | 145.61 | 296.59 | 186.10 | 42.11 | 50.35 | 280.98 |
| 2002 analysis | 285.54 | 29.26 | 194.00 | 308.00 | 263.90 | 23.23 | 199.00 | 288.00 | 203.93 | 40.87 | 0.00 | 276.00 |
| Difference | -3.496 | 25.084 | -110.608 | 58.498 | -14.077 | 26.035 | -142.387 | 49.425 | -17.827 | 51.348 | -217.562 | 160.815 |



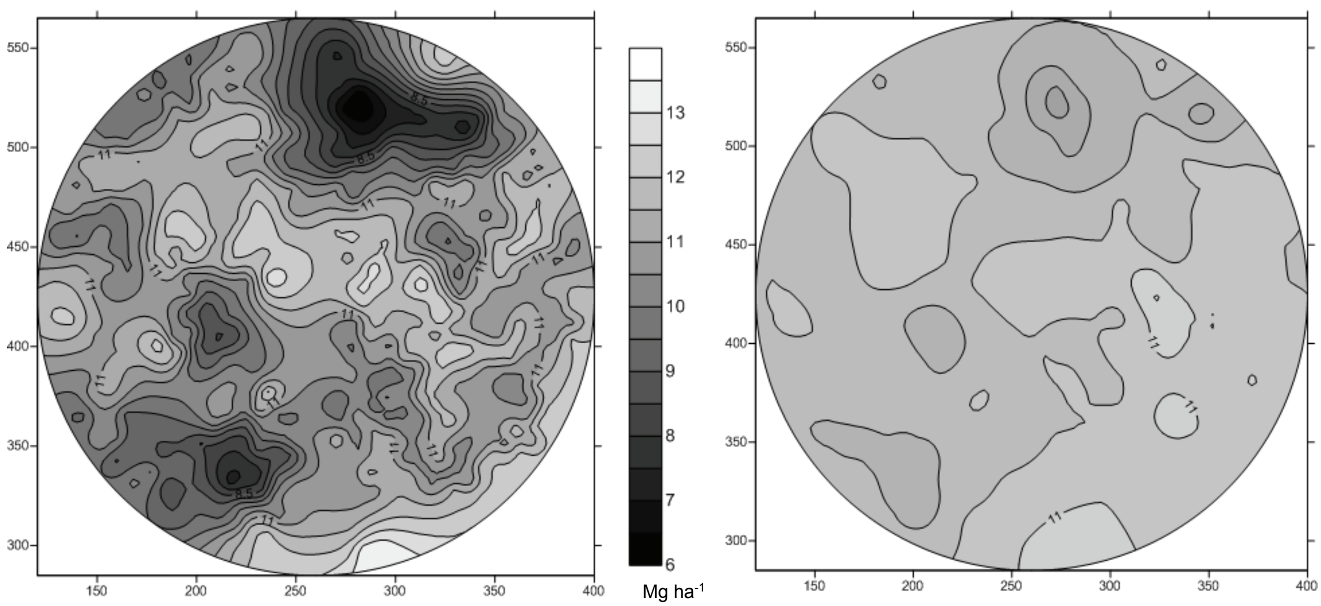

**(a)** **(b)**

**Figure 7. Rainfed estimated yield maps for the (a) 2014 analysis and (b) 2002 analysis for 1999.**




(a) (b)

**Figure 8. Rainfed estimated yield maps for the (a) 2014 analysis and (b) 2002 analysis for 2000.**




(a) (b)

**Figure 9. Maximum yield estimate yield maps for the (a) 2014 analysis and (b) 2002 analysis for 2000.**

**Table 2. Agreement between Bayesian (2014 analysis) and Sadler et al. (2002) (2002 analysis) using regression analysis.**

| Year | Variable | Intercept | Slope | $r^2$ |
|---|---|---|---|---|
| 1999 | Rainfed yield | 2.04 | 0.65 | 0.81 |
| | Maximum yield | 6.11 | 0.43 | 0.60 |
| | Irrigation at maximum yield | 110.89 | 0.62 | 0.43 |
| | Response to irrigation | 2.10 | 0.55 | 0.61 |
| 2000 | Rainfed yield | 3.46 | 0.32 | 0.57 |
| | Maximum yield | 7.90 | 0.24 | 0.54 |
| | Irrigation at maximum yield | 165.23 | 0.39 | 0.19 |
| | Response to irrigation | 4.15 | 0.21 | 0.43 |
| 2001 | Rainfed yield | 6.04 | 0.41 | 0.54 |
| | Maximum yield | 9.03 | 0.22 | 0.39 |
| | Irrigation at maximum yield | 161.58 | 0.23 | 0.05 |
| | Response to irrigation | 0.75 | 0.42 | 0.43 |

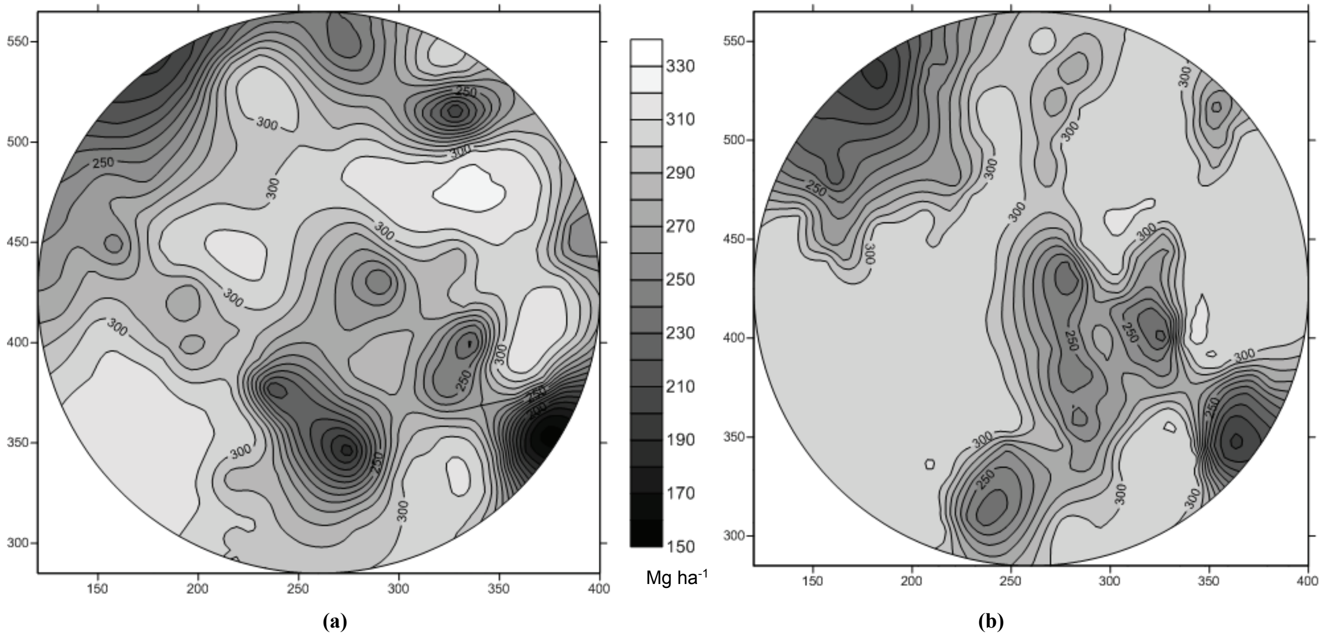



**Figure 10. Irrigation at maximum yield estimate maps for the (a) 2014 analysis and (b) 2002 analysis for 1999.**

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