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Estimating In Situ Unsaturated Hydraulic Properties of Vertically Heterogeneous Soils

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ABSTRACT

The majority of procedures for in situ measurement of the unsaturated hydraulic conductivity are variations of the instantaneous profile method. A vertically nonuniform soil requires the unsaturated hydraulic functions to be estimated at each horizon. Scaling systems have evolved in an attempt to reduce the number of hydraulic functions needed to characterize water flow through heterogeneous soils. In this study, we extended the concept of water content (θ) scaling to nonuniform soil profiles, tested the effectiveness of θ scaling for reducing apparent spatial variability, and estimated the unsaturated hydraulic functions for a naturally occurring loamy sand field site. Two instantaneous profile experiments conducted at Etiwanda, CA, provided soil water content and pressure head (h) data vs. depth (z) and time (t). Water retention, $\theta(h)$, and hydraulic conductivity, $K(\theta)$, functions fitted to data from the 15-cm depth at Plot 1 were arbitrarily chosen as the reference hydraulic properties to which the other depths and plots were scaled. Based on a unit-gradient analysis of the drainage data, the slope of the hydraulic conductivity function, $dK/d\theta$, was estimated as z/t. Scaling other depths and plots to the reference location was done using an iterative procedure that provided least-squares estimates of the two θ scaling parameters (δ and μ) and a corresponding transformed depth variable (z^*). Scaled water content, θ^*, plotted vs. z^*/t, using data from all depths and plots, coalesced to a single curve. Scaling θ successfully coalesced heterogeneous soil hydraulic properties into unique functions for both $\theta(h)$ and $K(\theta)$.

THE MAJORITY OF METHODOLOGIES for field measurements of the unsaturated soil hydraulic properties are limited to variations of the instantaneous profile method (Green et al., 1986; Rose et al., 1965). The instantaneous profile method requires that θ and/or pressure head (h) measurements be made at frequent intervals of t and z. Therefore, the procedure is both time consuming and labor intensive. Also, calculated $K(\theta)$ values are directly influenced by the precision of the field instruments used, and by the numerical estimation of derivatives via differencing of θ and h measurements (Flühler et al., 1976). Smoothing field-measured $\theta(t)$ and $h(t)$ data may reduce differencing errors (Ahuja et al., 1980); however, as noted by Sisson and van Genuchten (1991), smoothing is subjective and its influence on hydraulic conductivity estimates is not well documented.

Natural variability of field soils further confounds the problems associated with measuring $K(\theta)$. Generally, a large number of model parameters are necessary to fully characterize a nonuniform soil profile. Complete characterization requires measurements to be taken at each horizon and site of interest, leading to site- and depth-specific hydraulic functions.

To minimize the difficulty and cost of characterizing water flow through heterogeneous field soils, scaling approaches have evolved in an attempt to reduce the number of hydraulic functions needed (Miller and Miller, 1956; Warrick et al., 1977; Simmons et al., 1979; Ahuja et al., 1984; Tillotson and Nielsen, 1984). By defining scaling relationships, the number of parameters necessary to describe the hydraulic properties of a spatially variable field may be greatly reduced. Sposito and Jury (1985) generalized the concept of scaling water contents in the Richards equation using a linear relationship. Sisson (1987) developed a water content scaling system for layered soils but the scaling factors in his fixed-gradient analysis depended on the mathematical form used to represent the hydraulic conductivity function.

The objectives of this study were to: (i) extend θ

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scaling to vertically heterogeneous soils by allowing the scale factors to vary with depth; (ii) test if linear θ scaling can reduce the apparent spatial variability of soil hydraulic functions; and (iii) illustrate the method by determining the unsaturated hydraulic properties for a naturally occurring soil. The scale factors in our θ scaling procedure are independent of the form of the hydraulic conductivity function. The method requires one set of reference hydraulic functions for water retention and hydraulic conductivity, as well as two scale factors for each horizon of interest in the profile.

THEORY

Scaling Relationships

The Richards equation may be used to describe one-dimensional vertical water flow in a rigid, vertically nonuniform soil profile:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(z,\theta)\frac{\partial h(z,\theta)}{\partial z}\right] - \frac{\partial K(z,\theta)}{\partial z} \quad [1]$$

where $\theta(z,t)$ is the volumetric water content (m^3/m^3), t is time (h), z is depth (cm), $h(z,\theta)$ is the pressure head (cm), and $K(z,\theta)$ is the hydraulic conductivity (cm/h).

The fundamental idea behind our scaling procedure is to transform Eq. [1] into a form such that the hydraulic functions are invariant with depth (i.e., the soil profile becomes homogeneous):

$$\frac{\partial \theta^*}{\partial t} = \frac{\partial}{\partial z^*}\left[K^*(\theta^*)\frac{\partial h^*(\theta^*)}{\partial z^*}\right] - \frac{\partial K^*(\theta^*)}{\partial z^*} \quad [2]$$

where the notation * indicates a scaled parameter. The transformation involves a simple change of variables using the chain rule of calculus:

$$\begin{aligned}\frac{\partial \theta}{\partial t} &= \frac{\partial \theta}{\partial \theta^*}\frac{\partial \theta^*}{\partial t} \\ &= \frac{\partial z^*}{\partial z}\frac{\partial}{\partial z^*}\left(K(z,\theta)\frac{\partial h}{\partial h^*}\frac{\partial h^*}{\partial z^*}\frac{\partial z^*}{\partial z}\right) \\ &\quad - \frac{\partial z^*}{\partial z}\frac{\partial K(z,\theta)}{\partial K^*(\theta^*)}\frac{\partial K^*(\theta^*)}{\partial z^*}\end{aligned} \quad [3]$$

which can be rewritten as

$$\frac{\partial \theta^*}{\partial t} = \omega\frac{\partial}{\partial z^*}\left[K(z\theta)\beta\frac{\partial h^*}{\partial z^*}\right] - \xi\frac{\partial K^*(\theta^*)}{\partial z^*} \quad [4]$$

where ω, β, and ξ are defined as transformation parameters

$$\omega = \frac{\partial \theta^*}{\partial \theta}\frac{\partial z^*}{\partial z}, \quad [5]$$

$$\beta = \frac{\partial h}{\partial h^*}\frac{\partial z^*}{\partial z}, \quad [6]$$

and

$$\xi = \frac{\partial \theta^*}{\partial \theta}\frac{\partial z^*}{\partial z}\frac{\partial K(z,\theta)}{\partial K^*(\theta^*)}. \quad [7]$$

There are many ways to transform Eq. [1] into Eq. [2]. We will restrict the solution set such that the soil water flux is invariant under the transformation, and require volumetric water contents to scale according to the simple linear relationship:

$$\theta^*(z^*,t) = \delta(z) + \mu(z)\theta(z,t) \quad [8]$$

in which δ and μ are depth-dependent scaling factors. Note that we decided not to scale time (i.e., $t = t^*$). Equation [8] implies that, for each soil layer, two parameters (δ and μ) can be found that relate the local water content to the scaled water content at any time t. Differentiating Eq. [8] with respect to θ yields

$$\frac{\partial \theta^*}{\partial \theta} = \mu(z). \quad [9]$$

We now require $\omega = 1$ in Eq. [5]. This assumption implies that, in our scaled system, z^* is independent of time as would be expected for a scaled spatial coordinate. Substituting Eq. [9] into Eq. [5] leads to

$$\left(\frac{\partial z^*}{\partial z}\right)_t = \frac{dz^*}{dz} = \frac{1}{\mu(z)}. \quad [10]$$

Integrating Eq. [10] with respect to depth gives

$$z^* = \int_0^z \frac{dz'}{\mu(z')} \quad [11]$$

where we have assumed that $z^* = 0$ at the soil surface.

As a second requirement of our scaling method, we set

$$K(z,\theta) = K^*(\theta^*). \quad [12]$$

To examine the implications of this requirement, it may be helpful to consider a simple functional form for $K(\theta)$. For example, let us use the Davidson model (Davidson et al., 1969):

$$K^j(\theta^j) = K_s^j \exp^{[\sigma^j(\theta^j - \theta_s^j)]} \quad [13a]$$

In Eq. [13a], σ is an empirical parameter, the subscript s denotes the value at saturation of the indicated parameter and the superscript j is either * or z, representing the scaled space or any arbitrary location in real space, respectively. Substituting Eq. [13a] in the left- and right-hand sides of Eq. [12], which represent the scaled (*) and unscaled (superscript z) spaces, respectively, and solving for θ^* leads to

$$\theta^*(z^*,t) = a(z) + b(z)\theta^z(z,t) \quad [13b]$$

where $$a(z) = \theta_s^* + \frac{1}{\sigma^*}\ln\left(\frac{K_s^z}{K_s^*}\right) - \frac{\sigma^z}{\sigma^*}\theta_s^z$$

and $$b(z) = \frac{\sigma^z}{\sigma^*}.$$

Equation [13b] provides a convenient way of implementing

Eq. [8] when the Davidson model is used for the hydraulic conductivity function. For other functional forms, Eq. [8] should be used directly in conjunction with Eq. [12] to give an approximate relationship between θ and θ^*.

The requirement that soil water flux is invariant under the transformation, coupled with Eq. [12], implies that the scaled hydraulic head (H^*) satisfies

$$\frac{\partial H}{\partial z} = \frac{\partial H}{\partial H^*}\frac{\partial H^*}{\partial z^*}\frac{\partial z^*}{\partial z} = \frac{\partial H^*}{\partial z^*}, \qquad [14]$$

or with Eq. [10]

$$\left(\frac{\partial H}{\partial H^*}\right)_t = \mu(z). \qquad [15]$$

If we assume that the surface boundary conditions of the scaled and unscaled soil profiles are the same [i.e., $h(0,t) = h^*(0,t)$], we can integrate Eq. [15] at a fixed instant in time, recalling that $dH = dh + dz$, which gives:

$$h^*(z,t) = h(0,t) + \int_{h(0,t)}^{h(z^*,t)} \frac{dh}{\mu(z)} \qquad [16]$$

where we have used Eq. [11]. Equation [16] provides an unambiguous definition of h^* for $h(z)$ profiles that can be represented as a single-valued differentiable function (such as during drainage or constant infiltration). The relationship also follows from Eq. [16] by assuming $\beta = 1$. The above requirements define one possible method of transforming Eq. [1] to Eq. [2]. Thus, we adopt Eq. [8] and [16], as well as the additional relationship $K^*(\theta^*) = K(z,\theta)$ as our scaling relations, with the scaled depth variable defined by Eq. [11]. We emphasize that this set of transformations forces the Richards equation for the nonuniform case to be written in a mathematically similar form as that for the uniform soil case.

METHODS

Experiment

The experimental part of the study was conducted at the Etiwanda field station near the University of California, Riverside. The soil at this site is classified as a Tujunga loamy sand mixed, thermic Typic Xeropsamment; Butters et al., 1989; Jury et al., 1982. Two 2 by 2 m plots were chosen that showed the most divergence in soil texture observed at the site (Jury et al., 1982). Plots 1 and 2 were instrumented with three and four polyvinyl chloride-plastic neutron access tubes, respectively. One access tube was located at the center of each 2 by 2 m bermed plot and the remaining tubes were located outside the plots. Plot 2 was also instrumented with one tensiometer at each 15-cm depth from 15 to 120 cm. Water was applied to the plots until neutron measurements in all access tubes remained unchanged for a period of 20 d (30 d of ponding total). The long flooding period helped to reduce lateral gradients in the soil water pressure potential. Neutron measurements and tensiometer readings began as soon as the ponded water infiltrated the surface. The plots were covered with both a plastic vapor barrier and thermal insulation to prevent evaporation from the soil surface. Tensiometer data were taken until the tensiometers failed after 2 h of drainage. Neutron probe data were taken until three consecutive weekly measurements were unchanged. At the end of the drainage experiment, the soil was excavated and undisturbed soil core samples (5-cm i.d. and 7 cm long) were taken at selected depths (see Table 1) within each soil profile. These cores were used to determine bulk density, soil particle-size distribution, and soil water retention in the laboratory using standard methods.

Table 1. Particle-size fractions and bulk densities for the Etiwanda field site, Plots 1 and 2.

	Depth increment	Coarse sand	Sand	Silt	Clay	Bulk density
	cm	g/g				Mg/m^3
Plot 1	0–15	0.01	0.86	0.09	0.04	1.51
	15–30	0.01	0.85	0.09	0.05	1.52
	30–60	0.02	0.89	0.06	0.03	1.49
	60–90	0.10	0.85	0.03	0.02	1.50
	90–120	0.01	0.89	0.07	0.03	1.54
Plot 2	0–15	0.02	0.74	0.17	0.07	1.44
	15–30	0.02	0.73	0.18	0.07	1.45
	30–45	0.03	0.72	0.18	0.07	1.45
	45–60	0.03	0.72	0.17	0.08	1.44
	60–75	0.04	0.72	0.18	0.06	1.45
	75–90	0.03	0.73	0.18	0.06	1.44
	90–105	0.02	0.73	0.18	0.07	1.43
	105–120	0.02	0.72	0.19	0.07	1.44
	120–135	0.02	0.74	0.17	0.07	1.45

Parameter Estimation

Initially at saturated water content, our field sites were covered and allowed to drain while measurements of volumetric water content and pressure head were made over time. Assuming that the Richards equation has been appropriately scaled from Eq. [1] to [2], it is reasonable to further assume that under our experimental conditions gravity is the dominant driving force for water flow during drainage, i.e., $\partial h^*(\theta^*)/\partial z^* = 0$ (Sisson, 1987; Chong et al., 1981). This assumption leads to the unit-gradient approximation of the Richards equation (written in scaled form):

$$\frac{\partial \theta^*}{\partial t} = \frac{\partial K^*(\theta^*)}{\partial z^*}. \qquad [17]$$

Sisson et al. (1980) presented an implicit solution of Eq. [16] for any monotonic increasing $K^*(\theta^*)$ function as:

$$\frac{dK^*}{d\theta^*} = \frac{z^*}{t}. \qquad [18]$$

Therefore, we can estimate $dK^*/d\theta^*$ from the ratio of scaled depth to time. The usefulness of Eq. [18] to the analysis of instantaneous profile experimental data can be fully realized by using analytical expressions for the hydraulic functions. In this study, we used the van Genuchten (1980) expressions for $\theta(h)$ and $K(\theta)$ as follows:

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + |\alpha h|^n)^m} \qquad [19]$$

and

$$K(\theta) = K_s S^\lambda[1 - (1 - S^{1/m})^m]^2. \qquad [20]$$

In the above equations the relative saturation (s) is defined as

$$S = \frac{\theta - \theta_r}{\theta_s - \theta_r}, \qquad [21]$$

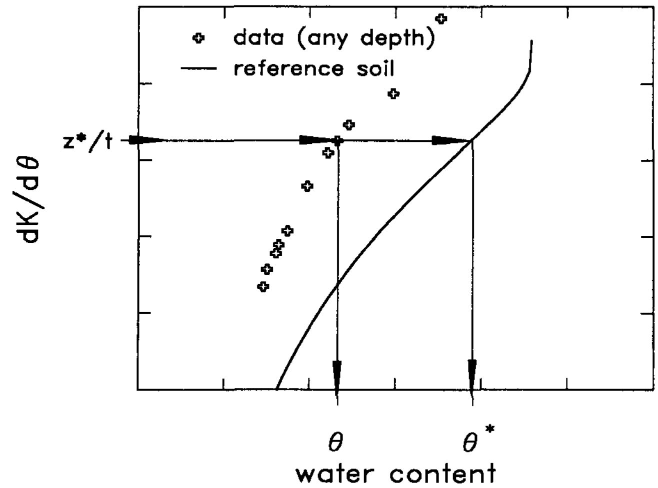

Fig. 1. Schematic of the procedure for obtaining estimates of water content, θ, and scaled water content, θ^*, during the iterative regression analysis.

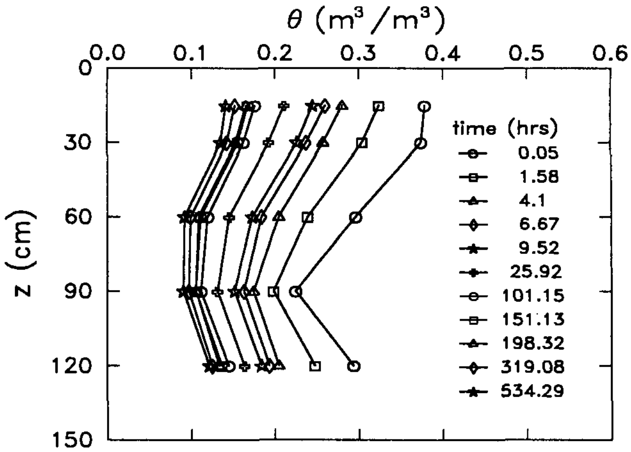

Fig. 2. Measured water contents, θ, vs. depth, z, for selected times during drainage of Plot 1.

λ is the pore connectivity parameter (Luckner et al., 1989), θ_s is the saturated water content, θ_r is the residual water content, K_s is the saturated hydraulic conductivity, α and n are shape factors, and $m = 1 - 1/n$.

Using the relationship given in Eq. [21], Sisson and van Genuchten (1991) differentiated Eq. [20] to obtain:

$$\frac{dK}{d\theta} = \frac{1}{(\theta_s - \theta_r)}\frac{dK}{dS} = \frac{K_s S^{\lambda-1}}{\theta_s - \theta_r}(1 - A^m)(\lambda + 2S^{1/m}A^{m-1} - \lambda A^m), \quad [22]$$

where

$$A = 1 - S^{1/m}. \quad [23]$$

We used their optimization program (UNGRA) with $dK^*/d\theta^*$ (i.e., z^*/t) data, as well as laboratory water retention data, to optimize the parameters needed to characterize the hydraulic properties of the reference soil (i.e., K_s^*, θ_s^*, θ_r^*, α, λ, and n). The advantage of using this method is that instantaneous profile data are formulated in terms of a parameter optimization, thus allowing the range of experimental data to be extended with measurements made independently of the drainage experiment. We arbitrarily chose the 15-cm depth of Plot 1 as our reference soil.

Scaling

The water content, depth, and time data were analyzed using an iterative regression technique to accomplish scaling of the two soil profiles. The following steps outline the analysis:

1. The 15-cm depth at Plot 1 was arbitrarily chosen to be the reference location for which $\theta = \theta^*$ and $z = z^*$. Using the Sisson and van Genuchten (1991) optimization program, UNGRA, the parameters K_s^*, θ_s^*, θ_r^*, α, λ, and n were estimated from laboratory water retention data and field-obtained values of z^*/t vs. θ^*. Using these estimated parameters for the scaled space, δ and μ were estimated for each remaining depth.
2. The scaling factors δ and μ needed at the remaining depths (and plot) were estimated iteratively:
   a. As a first approximation, z^* was set equal to z. For every measured z/t (and hence $dK/d\theta$) at any depth–plot combination, a value of θ^* was estimated from the functional form of $dK^*/d\theta^*$ using the scheme illustrated in Fig. 1. Scaling commences from the soil surface, as required by Eq. [11].
   b. For each depth, measured θ and estimated θ^* were paired and used in a linear regression to estimate δ and μ (the regression method assumed errors in θ and θ^* to be equal).
   c. Using the δ and μ values, z^* was recalculated using numerical integration of Eq. [11].

   The iterative procedure (Steps 2a–c) was terminated when successive estimates of z^* agreed to within 0.01 cm.
3. The next depth was selected and Step 2 repeated until all depths at both plots were scaled.

RESULTS AND DISCUSSION

Particle-size distributions and bulk densities for various depth increments in the two plots are given in Table 1. The first 15 to 30 cm of the Plot 1 profile

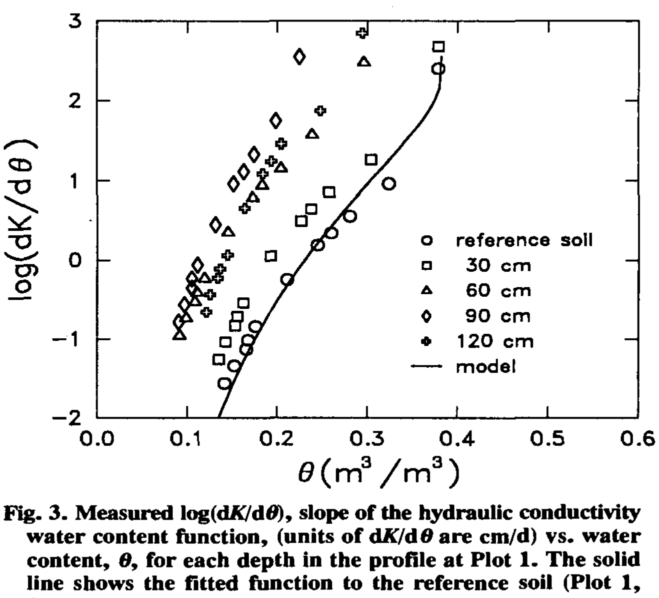

Fig. 3. Measured log($dK/d\theta$), slope of the hydraulic conductivity water content function, (units of $dK/d\theta$ are cm/d) vs. water content, θ, for each depth in the profile at Plot 1. The solid line shows the fitted function to the reference soil (Plot 1, 15-cm depth).

Table 2. Scaling parameters† for Etiwanda field soil, Plots 1 and 2.

	Depth	z^*	δ	μ	r^2
	cm	cm			
Plot 1	15	15.000	0.000	1.000	---
	30	29.998	0.034	1.000	0.984
	60	57.470	0.087	1.092	0.984
	90	76.916	0.053	1.543	0.981
	120	98.770	0.043	1.373	0.955
Plot 2	15	18.228	0.020	0.823	0.990
	30	37.491	0.041	0.779	0.986
	45	57.148	0.052	0.763	0.990
	60	76.478	0.051	0.776	0.991
	75	96.947	0.048	0.733	0.990
	90	118.227	0.048	0.705	0.986
	105	136.979	0.022	0.800	0.980
	120	154.350	0.005	0.864	0.966
	135	172.933	0.032	0.807	0.976

† z^* is the scaled depth, and δ and μ are depth-dependent scaling factors.

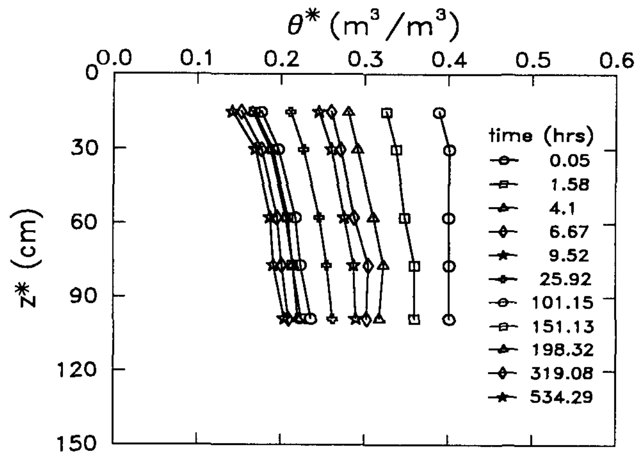

Fig. 4. Scaled water content, θ^*, vs. scaled depth, z^*, at selected times during drainage.

has a sandy to loamy sand texture, includes all of the Ap horizon, and is enriched with organic matter. The texture becomes slightly coarser with depth to ≈60 cm. There is a distinct coarse sand layer between 60 and 90 cm. Coarse sand and cobbles are present below 120 cm. The Plot 2 profile has a more uniform texture ranging from loamy sand to sandy loam and has a more uniform bulk density distribution than Plot 1, whereas the average silt and clay contents are about twice that of Plot 1. These two plots bracket the extremes in terms of observed texture, water content, and solute movement for the Etiwanda field site (Butters et al., 1990).

Figure 2 shows the in situ water contents of Plot 1 measured with a neutron probe during the drainage phase. Water contents ranged from 0.38 to 0.23 m^3/m^3 at field saturation across the 120-cm profile, which indicates substantial vertical variability in hydraulic and retention properties. More water was lost to drainage from the upper three measured depths than from the lower two depths, similar to the pattern expected for a profile with a constant gradient (Sisson, 1987). Water content distribution by depth at Plot 2 (not shown) showed less vertical heterogeneity, with θ_s ranging from 0.42 to 0.47 m^3/m^3.

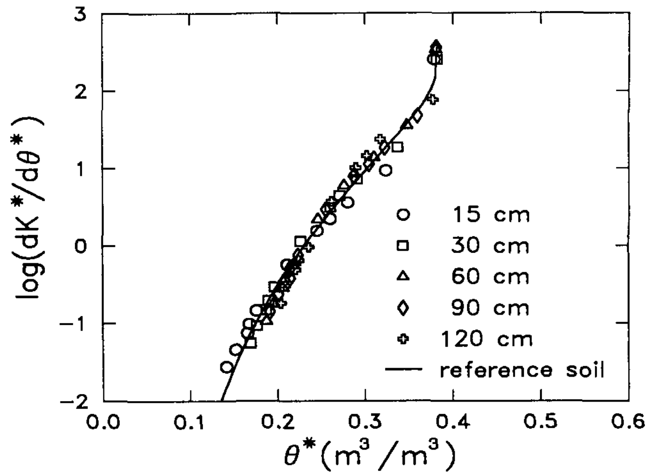

Fig. 5. Scaled ($dK/d\theta$), slope of the hydraulic conductivity water content function, (units for $dK^*/d\theta^*$ are cm/d) vs. scaled water content, θ^*, for Plot 1.

Figure 3 shows plots of the observed $dK/d\theta$ (as estimated from z/t) vs. water content for each depth at Plot 1. The figure also shows the fitted $dK/d\theta$ curve based on Eq. [19] and [22]. The large differences among the data at different depths indicates that several $dK/d\theta$ functions would have been required to describe this soil profile using conventional analysis.

Values of the empirical scaling parameters, δ and μ, and the r^2 values resulting from regressing θ^* vs. θ, are presented in Table 2. Table 2 shows that the scaling procedure compressed the soil profile at Plot 1 by ≈20 cm because of lower water contents in the deeper layers, compared with the reference soil. Scaling θ at Plot 2, on the other hand, resulted in an increased scaled depth ($z^* < 135$ cm) because of higher water contents in this profile compared with the reference soil. At Plot 2 the values of the scaling parameter μ ranged from 0.705 to 0.864, whereas μ ranged from 1.00 to 1.54 at Plot 1. The scaled water contents vs. depth shown in Fig. 4 are considerably more uniform with depth than those for the unscaled water contents in Fig. 2. This indicates, as expected, that the scaling method does reduce the apparent vertical heterogeneity in water content distribution within the profile.

While the water content scaling equations of Sposito and Jury (1985), including our Eq. [8], are similar to other relationships in the literature, there are several differences. For example, Simmons et al. (1979), Libardi et al. (1980), and Sisson et al. (1980) all noted that for a draining soil profile the average θ above a depth was linearly related to the θ at that depth. These results are not necessarily consistent with the equations of Sposito and Jury (1985) nor the relationships presented here. Actually, our scaling system is more similar to the method proposed by Warrick et al. (1977), who replaced θ with a saturation variable (θ/θ_s) in the Richards equation. Unfortunately, the resulting equation for water flow did not conserve mass. In contrast, our results can be viewed as using relative saturation for θ and scaling depth to ensure mass conservation.

A plot of $dK^*/d\theta^*$ (i.e., z^*/t) vs. θ^* values using

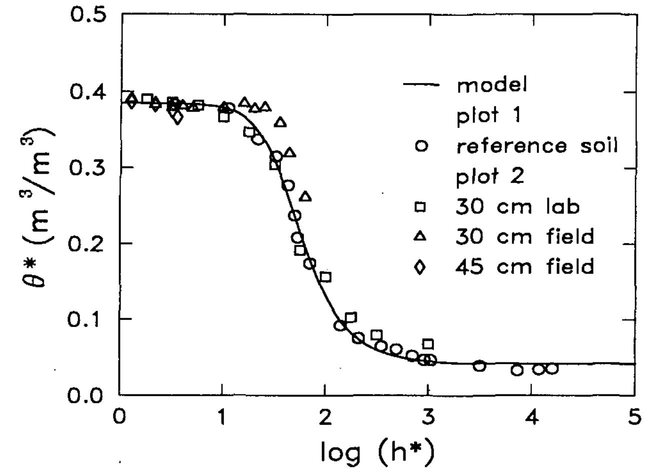

Fig. 6. Scaled soil water retention curve showing field- and laboratory-measured data and the modeled water retention. θ^* is the scaled water content and h^* is the scaled pressure head.

Table 3. Sisson-van Genuchten curve-fitting parameters† for the Etiwanda field soil at Plot 1, 0 to 15-cm depth increment.

θ_r	θ_s	α	n	K_s	λ	r^2
cm^3/cm^3	cm^3/cm^3			cm/h		
0.0416	0.3848	0.0243	2.4498	4.6467	3.9470	0.9980

† θ_r is the residual water content; θ_s is the saturated water content; α and n are shape factors; K_s is the saturated hydraulic conductivity; λ is the pore connectivity parameter.

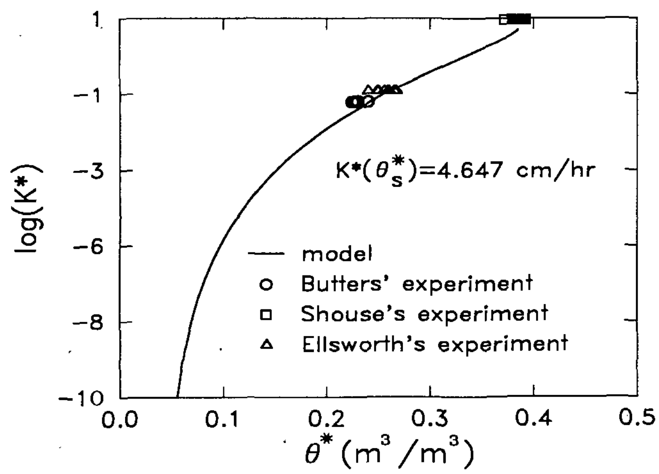

Fig. 7. Fitted hydraulic conductivity function for the reference soil (units for scaled hydraulic conductivity, K^*, are cm/d). Independently measured steady-state conductivities, after being scaled using the proposed scaling method, are also shown. θ^* is the scaled water content, θ_s^* is the scaled water content at saturation.

data from all depths at Plot 1 is given in Fig. 5. Comparison of Fig. 3 and 5 reveals that our linear scaling method has coalesced dK/dθ vs. θ into a single scaled function. Since Sisson and van Genuchten (1991) have shown that the hydraulic function parameters using dK/dθ were nearly identical to the parameters resulting from fitting K data, we conclude that linear scaling coalesced hydraulic conductivities as well.

In our present application of the proposed scaling procedure, the unit gradient approximation removes the need for scaling h. For illustrative purposes, however, the field pressure head measurements and the two laboratory-measured water retention curves (the referenced soil, i.e., the 15-cm depth at Plot 1, and the 30-cm depth from Plot 2) provide h data for scaling. Equation [16] was used to scale the field data, while the approximate relationship $h^* = h/\mu(z)$ was used for scaling measured water retention curves. Using these relationships, we scaled the available data as illustrated in Fig. 6. This figure shows the fitted (using the UNGRA program) water retention curve, laboratory-measured retention data from Plot 1, and scaled field and scaled laboratory data from Plot 2 (field pressure head data are limited because of equipment failure). The parameters for the fitted water retention curve shown in Fig. 6 are listed in Table 3. These parameters were derived from the simultaneous fit of z^*/t vs. θ^* and laboratory retention data from the 15-cm depth of Plot 1 (the reference soil).

Several other studies (e.g., Butters et al., 1989; Ellsworth et al., 1991) were conducted at the Etiwanda field site. As part of those studies, water contents at Plots 1 and 2 were measured during steady-state sprinkler irrigation. We scaled the measured water contents using the μs and δs listed in Table 2, thus effectively scaling the previously measured steady-state hydraulic conductivities. Figure 7 shows a comparison between the scaled hydraulic conductivity function estimated from data presented here and the steady-state K values measured by Butters et al. (1989) and Ellsworth et al. (1991). The Sisson and van Genuchten (1991) parameters for the fitted hydraulic conductivity function are listed in Table 3. The hydraulic conductivity functions estimated in this study from the measured properties of the reference soil (the 15-cm depth at Plot 1), shown as a solid line, closely agree with the independently measured (but scaled according to our scaling method) data. This indicates that our water content scaling procedure is a powerful tool to reduce the apparent spatial variability of soil hydraulic properties.

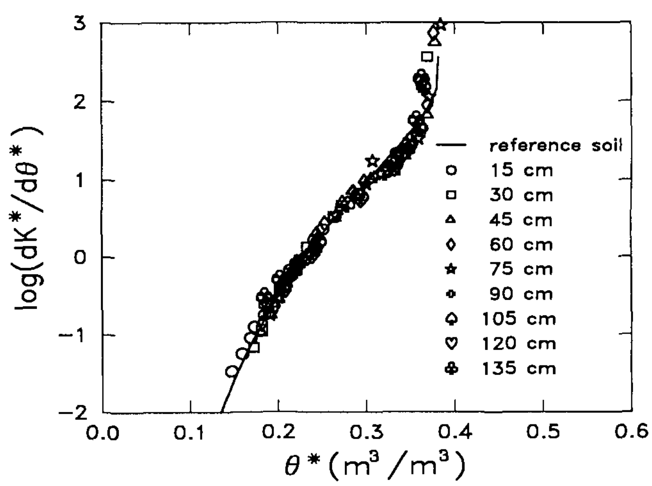

Fig. 8. Scaled dK/dθ, slope of the hydraulic conductivity water content function, (units for dK^*/dθ^* are cm/d) vs. scaled water content, θ^*, for Plot 2.

The results of scaling the $dK/d\theta$ vs. θ relationship at Plot 2 are shown in Fig. 8 and the associated scaling parameters are listed in Table 2. The profile was scaled to the reference soil. The solid line in the graph shows the $dK/d\theta$ function fitted to the reference soil. Clearly, our scaling method has coalesced the data from two distinct soil profiles with differing hydraulic properties. Also, the same scaling factors were used in scaling the retention function and the conductivity function. Other studies using similar media scaling concepts (e.g., Youngs and Price, 1981; Rao et al., 1983; Jury et al., 1987) have found that scaling factors for $K(\theta)$ are different than those for $\theta(h)$. Our results indicate that the number of fitting parameters necessary to characterize these heterogeneous profiles could be reduced drastically from 84 (fitting each layer separately) to 32 (fitting the scaled data).

SUMMARY

A linear scaling method was used to scale a multiplot vertically heterogeneous soil system into a soil with apparently uniform hydraulic properties. The linear scaling factors relating θ to θ^* depend on depth only. The resulting uniform profile required only six van Genuchten (1980) hydraulic parameters, plus two scaling factors per depth (32 values total), to fully describe the $K^*(\theta^*)$ and $h^*(\theta^*)$ functions. After scaling, all gravity-drainage (z/t) data could be used in the curve-fitting process if desired. In addition, the analysis of the instantaneous profile data has been cast in the form of an optimization problem that allows additional data, taken independently from the drainage experiment, to be included in the parameter estimation process.

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