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NEW MODELS FOR UNSATURATED SOIL HYDRAULIC PROPERTIES

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Two relatively simple models are proposed for describing the soil water retention curve. The expressions define sigmoidal or bimodal type retention functions with four or five parameters, respectively. The sigmoidal retention model may be combined with predictive pore-size distribution theories to yield closed-form equations for the unsaturated hydraulic conductivity. Parameters in the proposed hydraulic functions were estimated from observed retention data using a nonlinear least-squares optimization process. The models were tested on hydraulic data for more than 20 soils. Good agreement between predicted values and measured retention and conductivity data was found for most of the soils. The soil hydraulic models can be effectively utilized as inputs for numerical models of water flow and solute transport.

Computer models are widely used in research and management to predict water flow and solute transport in soils and groundwater. The accuracy of the predictions depends greatly on the reliability of the flow and transport properties of the medium being considered. Especially important in variably saturated flow studies are the unsaturated soil-hydraulic properties, i.e., the soil water retention and unsaturated hydraulic conductivity curves. As pointed out elsewhere (e.g., van Genuchten et al. 1991), several advantages exist for using relatively simple analytical expressions for the soil hydraulic properties.

Unsaturated hydraulic conductivity is an important soil property affecting the rate at which water and chemicals move through the vadose zone. However, its measurement is difficult, costly, time consuming, and frequently inaccurate. An alternative to direct measurements is the use of theoretical models to predict the unsaturated hydraulic conductivity from more easily measured soil properties such as the soil water retention curve. This indirect approach often provides an easy, efficient, and reasonably accurate way of estimating the conductivity function. Predictive conductivity models are generally based on statistical pore-size distribution theories, which assume that water flow through cylindrical pores can be described by the laws of Darcy and Poiseuille.

Many models for the retention and hydraulic conductivity functions have been developed during the past several decades. These include models by Childs and Collis-George (1950), Burdine (1953), Gardner (1958), Millington and Quirk (1961), Brooks and Corey (1964), Mualem (1976a), and van Genuchten (1980). While a large number of analytical soil retention functions have been proposed, only a few functions can be easily incorporated into predictive pore-size models (Mualem 1976a, 1986; van Genuchten et al. 1991) to yield relatively simple analytical expressions for the unsaturated hydraulic conductivity. Furthermore, most or all of retention models being used have S-shaped forms, which may fail to adequately characterize the retention curve of soils having multi-modal pore-size distributions (Othmer et al. 1991; Durner 1992). This problem has attracted recent attention because of the bimodal nature of many soil pore systems (Bouma 1981, 1984; Ross and Smettem 1993) and because of the importance of bimodal hydraulic functions in predicting preferential flow of water and chemicals in undisturbed soils or fracture rocks (Peters and Klavetter 1988; Gerke and van Genuchten 1993).

The objective of this paper is to present two relatively simple functions that can fit retention data exhibiting either a bimodal shape or a regular (unimodal) S shape. Closed-form expressions for the unsaturated hydraulic conductivity are derived for the S-shaped retention function using the predictive pore-size distribution models of Mualem (1976a) and Childs and Collis-George (1950). Finally, unsaturated hydraulic properties predicted with the proposed

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models are compared with observed data for different soils.

THEORY

Soil water retention functions

We propose the following equation for the description of water retention data of soils exhibiting bimodal pore-size distributions:

$$\theta(h) = \theta_r + (\theta_s - \theta_r)\,\frac{1 + c_1\alpha h}{1 + \alpha h + c_2(\alpha h)^2} \quad (1)$$

where θ is the volumetric water content, θ_r and θ_s are the residual and saturated water contents, respectively, h is the pressure head, α is a scaling factor, and c_1 and c_2 are empirical parameters affecting the shape of the retention curve. For notational convenience in this paper, we use absolute values for the negative pressure head (suction), h. To have a physically realistic curve (i.e., a monotonically decreasing function of θ with h) we impose the restrictions, $\alpha > 0$, $c_1 \geq 0$, and $c_2 \geq 0$, on the parameters in Eq. (1). Equation **(1)** can be expressed in dimensionless form as

$$S_e = \frac{1 + c_1 h^*}{1 + h^* + c_2 h^{*2}} \quad (2)$$

where the reduced pressure head, h^*, is given by

$$h^* = \alpha h \quad (3)$$

and the reduced water content (or effective saturation), S_e, is defined as

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (4)$$

Note that at saturation $h^* = 0$ and $S_e = 1$, while for very dry conditions $h^* \rightarrow \infty$ and S_e approaches 0. Figures 1 and 2 display relationships between the reduced pressure head and the reduced water content for different combinations **of** c_1 and c_2. Figure **1 shows** that, for $0 \leq c_1 \leq 1$, **the retention curve changes from an S-shape** function to a increasingly bimodal function when c_2 decreases. If $c_1 > 1$, $S_e(h^*)$ is no longer a monotonically decreasing function (Fig. 2 with $c_1 = 1.2$), and the retention curve becomes physically unrealistic. Therefore, only values between 0 and 1 will be selected for c_1.

The inverse relationship of the retention function, $h^*(S_e)$, yields two roots. Only the positive root

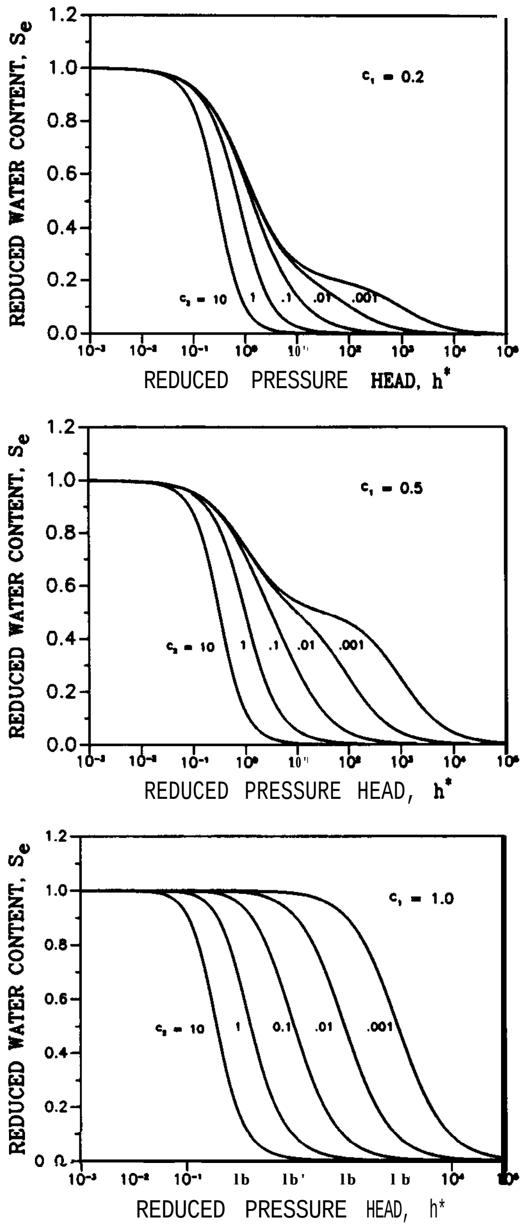

FIG. 1. **Soil water retention curves based on** Eq. (1) **for various values of** c_2 **and assuming (a)** $c_1 = 0.2$, (b) $c_1 = 0.5$, and (c) $c_1 = 1.0$.

$$h^* = \frac{(c_1 - S_e) + \sqrt{(c_1 - S_e)^2 + 4c_2 S_e(1 - S_e)}}{2c_2 S_e} \quad (5)$$

will ensure that $h^* = 0$ if $S_e = 1$, $h^* > 0$ for $0 < S_e < 1$, and $h^* \rightarrow \infty$ if $S_e = 0$.

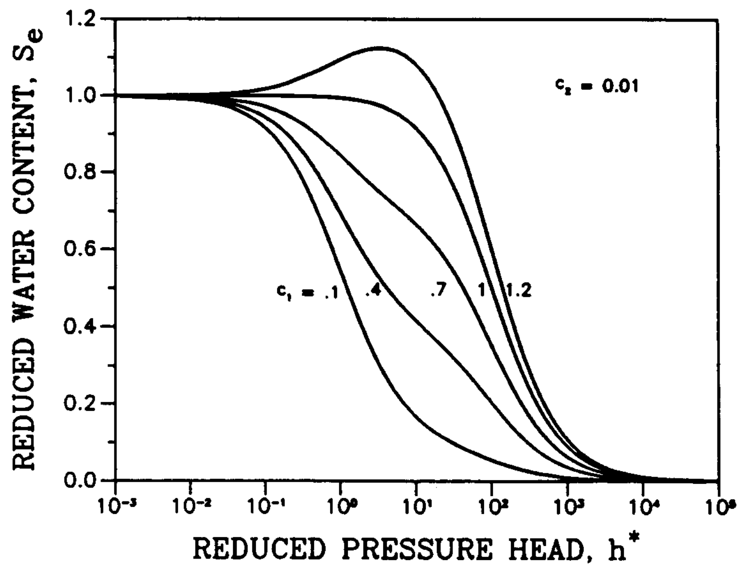

FIG. 2. Soil water retention curves based on Eq. (1) for various values of c_1 and assuming $c_2 = 0.01$.

By setting $c_1 = 1$ in Eq. (2), the bimodal retention function transforms into an expression for retention data exhibiting a sigmoidal distribution:

$$S_e = \frac{1 + h^*}{1 + h^* + c_2 h^{*2}} \tag{6}$$

with the inverse relationship

$$h^* = \frac{(1-S_e) + \sqrt{(1-S_e)^2 + 4c_2 S_e(1-S_e)}}{2c_2 S_e} \tag{7}$$

Hydraulic conductivity models

The model of Mualem (1976a) for predicting the unsaturated hydraulic conductivity function from soil water retention data is of the form

$$K(S_e) = K_s S_e^l [f(S_e)/f(1)]^2 \tag{8}$$

in which K and K_s are the unsaturated and saturated hydraulic conductivities, respectively, l is a pore connectivity or tortuosity parameter related to soil type, and

$$f(S_e) = \int_0^{S_e} \frac{1}{h(x)}\, dx \tag{9}$$

where S_e is given by Eq. (4).

The sigmoidal retention function given by Eq. (7) is particularly suitable for substitution in Mualem's model to obtain closed-form hydraulic conductivity functions. Substituting Eq. (7) into Eq. (9) gives

$$\begin{aligned} f(S_e) &= \int_0^{S_e} \frac{2\alpha c_2 x}{(1-x) + \sqrt{(1-x)^2 + 4c_2 x(1-x)}}\, dx \\ &= -\frac{\alpha}{2} \int_0^{w.f.} \frac{(1-x) - \sqrt{(1-x)^2 + 4c_2 x(1-x)}}{(1-x)}\, dx \end{aligned} \tag{10}$$

Let $y = 1 - x$, in which case Eq. (10) becomes

$$f(S_e) = \frac{\alpha}{2} \int_1^{1-w.f.} \frac{y - \sqrt{(1 - 4c_2)y^2 + 4c_2 y}}{y}\, dy \tag{11}$$

Integrating Eq. (11) (Gradshteyn and Ryzhik 1965) leads to

$$\begin{aligned} f(S_e) = {} & \frac{\alpha}{2}(1 - S_e) \\ & - \frac{\alpha}{2}\sqrt{(1 - S_e)(1 - S_e + 4c_2 S_e)} \\ & + \frac{\alpha c_2}{\sqrt{4c_2 - 1}} \\ & \cdot \left(\arcsin \frac{1 - 2c_2 - S_e + 4c_2 S_e}{2c_2} \right. \\ & \left. - \arcsin \frac{1 - 2c_2}{2c_2}\right) (c_2 > 0.25) \end{aligned} \tag{12}$$

$$f(S_e) = \frac{\alpha}{2}(1 - S_e) - \frac{\alpha}{2}\sqrt{(1 - S_e)(1 - S_e + 4c_2S_e)} + \frac{\alpha c_2}{\sqrt{1 - 4c_2}} \ln \frac{\sqrt{1 - 4c_2} + 1 - 2c_2}{\sqrt{(1 - 4c_2)(1 - S_e)(1 - S_e + 4c_2S_e)} + 1 - 2c_2 - S_e + 4c_2S_e} \quad (c_2 < 0.25) \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (8) gives expressions for the unsaturated hydraulic conductivity in conjunction with retention Eq. (6).

If $c_2 = 0.25$, the conductivity function reduces to a particularly simple form

$$K(S_e) = K_sS_e^l[S_e + 2(\sqrt{1 - S_e} - 1)]^2 \approx K_sS_e^{l+4}(0.25 + 0.75S_e)^2 \tag{14}$$

If $c_2 = 0.5$, another simple conductivity function results:

$$K(S_e) = \frac{4}{\pi^2}K_sS_e^l[1 - S_e - S_e\sqrt{1 - S_e} + \arcsin S_e]^2 \tag{15}$$

The expressions for $K(S_e)$ for other values of c_2 are somewhat more complicated (Eqs. (12) and (13)), but still can be evaluated readily.

Another predictive conductivity model used here is based on the theory by Childs and Collis-George (Childs and Collis-George 1950; Mualem 1986) as follows

$$K(S_e) = K_sS_e^lf(S_e)/f(1) \tag{16}$$

in which

$$f(S_e) = \int_0^{S_e} \frac{S_e - x}{h^2(x)}\,dx \tag{17}$$

Substituting Eq. (7) into Eq. (17) gives

$$f(S_e)/\alpha^2 = \frac{2c_2 - 1}{4(1 - 4c_2)} + c_2S_e + \frac{(1 + 2c_2)}{4}S_e^2 - c_2(1 - S_e)\ln(1 - S_e) + \left[\frac{1 - 2c_2}{4(1 - 4c_2)} - \frac{S_e}{4}\right]\sqrt{(1 - S_e)(1 - S_e + 4c_2S_e)} + \frac{c_2(1 - 5c_2 - S_e + 4c_2S_e)}{(1 - 4c_2)\sqrt{1 - 4c_2}} \ln \frac{\sqrt{(1 - 4c_2)(1 - S_e)(1 - S_e + 4c_2S_e)} + 1 - 2c_2 - S_e + 4c_2S_e}{\sqrt{1 - 4c_2} + 1 - 2c_2} \quad (c_2 < 0.25) \tag{18}$$

$$f(S_e)/\alpha^2 = \frac{2c_2 - 1}{4(1 - 4c_2)} + c_2S_e + \frac{1 + 2c_2}{4}S_e^2 - c_2(1 - S_e)\ln(1 - S_e) + \left[\frac{1 - 2c_2}{4(1 - 4c_2)} - \frac{S_e}{4}\right]\sqrt{(1 - S_e)(1 - S_e + 4c_2S_e)} + \frac{2c_2(1 - 5c_2 - S_e - 4c_2S_e)}{(4c_2 - 1)\sqrt{4c_2 - 1}} \left(\arcsin\frac{1 - 2c_2 - S_e + 4c_2S_e}{2c_2} - \arcsin\frac{1 - 2c_2}{2c_2}\right) \quad (c_2 > 0.25) \tag{19}$$

If $c_2 = 0.25$, a fairly simple expression for the hydraulic conductivity can be obtained as follows

$$K(S_e) = K_sS_e^l\left[1 + \frac{6}{5}(1 - S_e)\ln(1 - S_e) + \frac{6}{5}(1 - S_e) - \frac{2}{5}(1 - S_e)^{2/3} - \frac{9}{5}(1 - S_e)^2\right] \tag{20}$$

which is closely approximated by

$$K(S_e) \approx K_sS_e^{l+1}(1.467 - 1.156S_e + 0.239S_e^2 + 0.450S_e^3) \tag{21}$$

The conductivity expressions above assume that $c_1 = 1$ in Eq. (2). Similar exercises of integration can be carried out for other values of c_1 to compute $f(S_e)$ in the conductivity models given by Eqs. (8) and (16). Unfortunately, when this is done, $f(S_e)$ approaches infinity at $S_e = 1$. This shows that retention functions given by Eq. (2) with $c_1 < 1$ cannot be used to directly derive closed-form equations for the hydraulic conductivity according to the pore-size distribution theories by Mualem or Childs and Collis-George. This incompatibility of Eq. (2) with $c_1 < 1$ with the predictive conductivity theories is related to the value of the soil water capacity function $C(h) = d\theta/dh$, which becomes zero at saturation only when $c_1 = 1$. As pointed out by Nielsen and Luckner (1992), C(h) must, at a minimum, be zero at $h = 0$, and perhaps also approach zero asymptotically when $h \rightarrow 0$.

Rather than linking Eq. (2) directly with one of the predictive conductivity models, one could follow an alternative and more pragmatic approach by combining Eq. (2) with other empirical conductivity functions in order to characterize the soil hydraulic properties. One attractive, yet relatively simple, conductivity model for this purpose is the equation

$$K = K_s S_e \tag{22}$$

MATERIALS AND METHODS

Several soil hydraulic data sets, mainly from the catalog of Mualem (1976b), were used to test the hydraulic models above. Other data sets used included those from Moore (1939) and Jacobsen (1992, unpublished data). Soil texture of the soils ranged from sandy loam to clay. The selected data sets involved soil water retention and/or hydraulic conductivity measurements.

The model parameter vector, $b = (\theta_r, \theta_s, c_1, a, c_2, 1, K_s)$, was estimated using a nonlinear optimization approach which fits the retention and hydraulic conductivity functions to the observed retention and/or hydraulic conductivity data. Based on Marquardt's maximum neighborhood method (Marquardt 1963), a least-squares procedure was utilized to minimize the objective function, SSE(b), as follows

$$SSE(b) = \sum_{i=1}^{N} [Y_i - Y_i^*(\boldsymbol{b})]^2 \tag{23}$$

where SSE is the sum of squared error, and Y_i and Y_i^* are the observed and fitted retention and/or conductivity data, respectively. A detailed discussion of the optimization procedure is given by Kool et al. (1987) and van Genuchten et al. (1991).

RESULTS

Figures 3a and b show fitted curves according to Eq. (1) and observed retention data of a Danish soil (Tystofte) (Jacobsen 1992, unpublished data) and a Silt Mont soil (Mualem 1976b), respectively. The proposed model provides an excellent description of the retention data. Notice the bimodal nature of the curves in Fig. 3, especially those for the Tystofte soil. Durner (1992) and Othmer et al. (1991) previously showed that bimodal retention functions can be described by summing two sigmoidal retention curves given by van Genuchten's

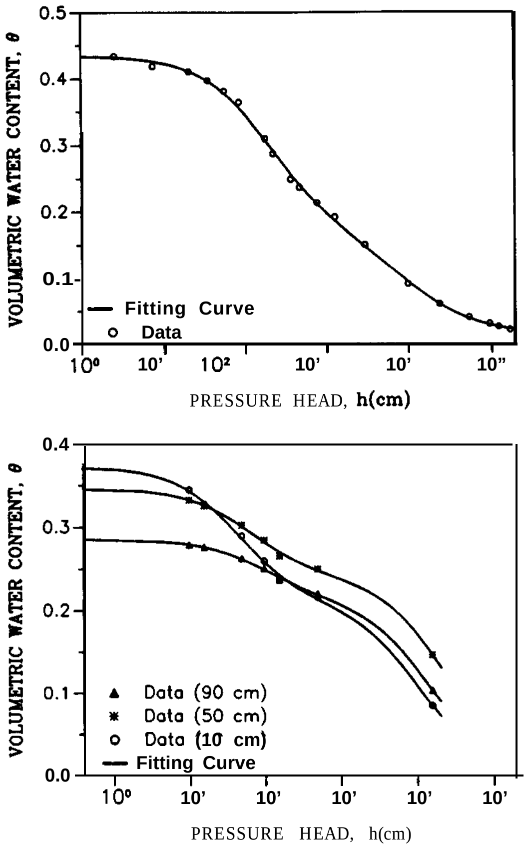

FIG. 3. Retention curves based on Eq. (1) fitted to observed retention data of (top) Silt Mont Cenis soil and (bottom) a Danish soil at three depths.

(1980) expressions, i.e.,

$$\theta_i(h) = \theta_{r,i} + (\theta_{s,i} - \theta_{r,i}) \frac{1}{[1 + (\alpha_i h)^{n_i}]^{1-1/n_i}} \quad i = 1, 2 \tag{24}$$

where θ_r and θ_s are the residual and saturated water contents, respectively, and α and n are empirical constants affecting the shape of the retention curve. Figure 4 gives an example of such a bimodal retention curve generated by two sigmoidal retention curves. The following parameters were used for this example: $\theta_{r,1} = 0.1$, $\theta_{s,1} = 0.35$, $\alpha_1 = 0.005$, $n_1 = 1.5$; $\theta_{r,2} = 0$, $\theta_{s,2} = 0.15$, $\alpha_2 = 0.15$, $n_2 = 2.5$. The curve in Fig. 4 can be described equally well with Eq. (1), but now with only five fitting parameters ($\theta_r = 0.109$, 8, $= 0.504$, $\alpha = 0.510$, $c_1 = 0.0638$, $c_2 = 0.0151$) instead of the eight used to generate the curve.

Next, the retention function given by Eq. (6) and the corresponding conductivity functions (Eqs. (12) to (15)) were used to describe a variety of soil data sets listed in Mualem (1976b). Observed and fitted retention curves are presented in Fig. 5a, b, and c for Sarpy Loam, Silt Loam GE3, and Yolo Light Clay, respectively. The parameters K_s and l in Eq. (8) were estimated from measured unsaturated conductivity data, using the nonlinear least-squares optimization method. Figure 6 shows a good agreement between the fitted hydraulic conductivity curve and the observed data for Silt Loam GE3. Figures Fig. 7a and b similarly present observed and fitted values of the unsaturated hydraulic conductivity as a function of water content for Sarpy Loam and Yolo Light Clay, respectively. Relatively good agreement was obtained, especially for Yolo Light clay.

Twenty-five hydraulic data sets of soils from

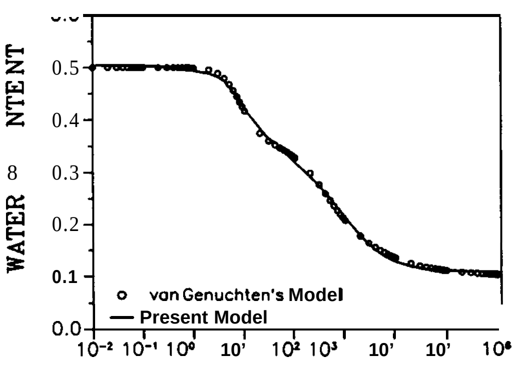

FIG. 4. Retention function according to Eq. (1) fitted to a bimodal data generated by a summation of two sigmoidal curves of van Genuchten.

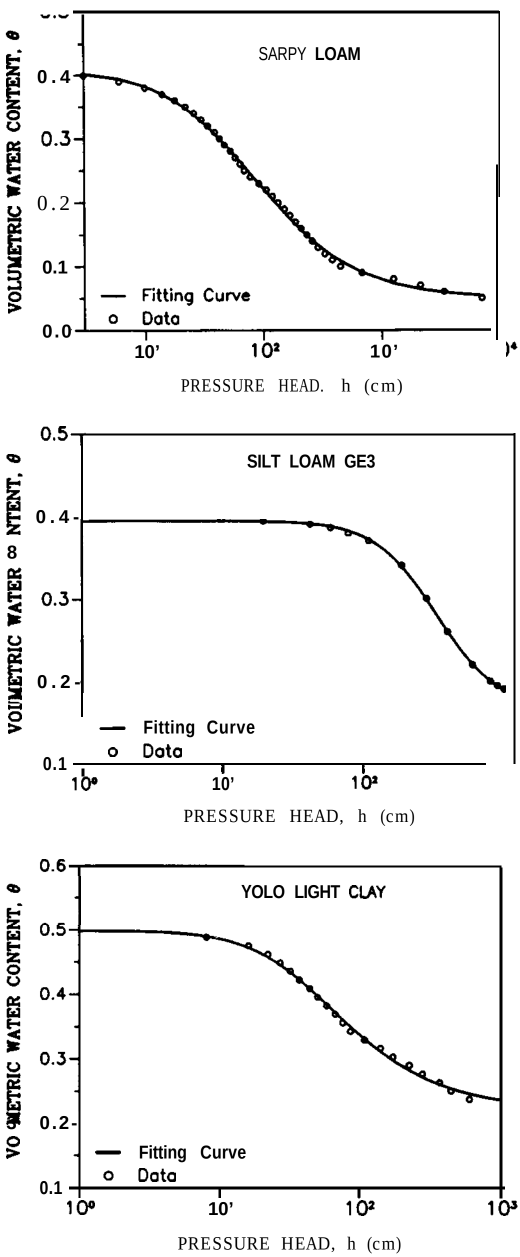

FIG. 5. Fitting retention curves based on Eq. (6) for (a) Sarpy Loam, (b) Silt Loam GE3, and (c) Yolo Light Clay.

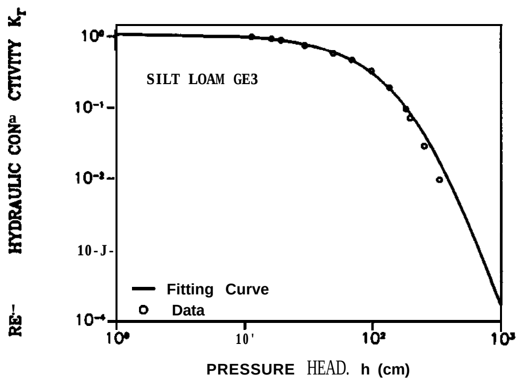

FIG. 6. Calculated hydraulic conductivity vs. pressure head based on Eqs. (12) through (15) for Silt Loam GE3.

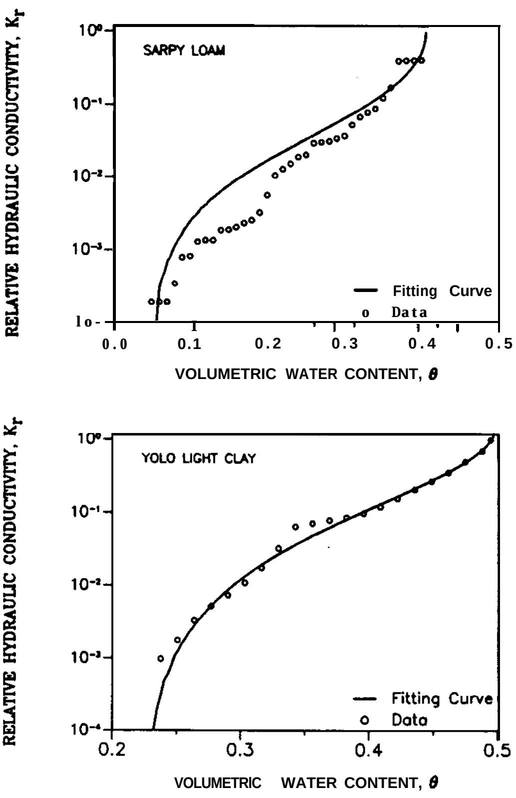

FIG. 7. Calculated hydraulic conductivity vs. water content based on Eqs. (12) to (15) for soils (a) Sarpy Loam, and (b) Yolo Light Clay.

Mualem's catalogue were subsequently used to compare the accuracy of Eq. (6), in combination with Eqs. (12) to (15), with the models developed previously by van Genuchten (1980) for retention and conductivity data. Through the optimization procedure, a sum of squared error (SSE) was estimated using Eq. (23) for each retention and conductivity data set. Figure 8 shows a scattergram of the calculated *SSE's*. The values of the *SSE* based on the proposed retention function (Eq. (6)) were quite similar to those obtained with van Genuchten's models.

Table 1 presents the fitted parameters of retention and hydraulic conductivity of 18 soils taken from the Mualem's catalogue. We had some convergence problems during the parameter estimation process for seven of the 25 soils. These seven data sets were not further considered in our analysis. The parameter values listed in Table 1 pertain to the retention model given by Eq. (6) and the conductivity form of Childs and Collis-George, i.e., Eqs. (18) through (20). For about half the soils, the fitted value of the exponential 1 were at or close to zero, consistent with the studies of Child and Collis-George (1950) and Mualem (1986). For most of the soils, the estimated values of the dimensionless saturated hydraulic conductivity were very close to the normalized value of 1. The table also contains fitting parameters for the wetting and drying branches of some retention and conductivity data. As shown by the relatively high coefficients of determination (r^2), the proposed hydraulic functions provided reasonably accurate descriptions of the hydraulic properties of the different soils.

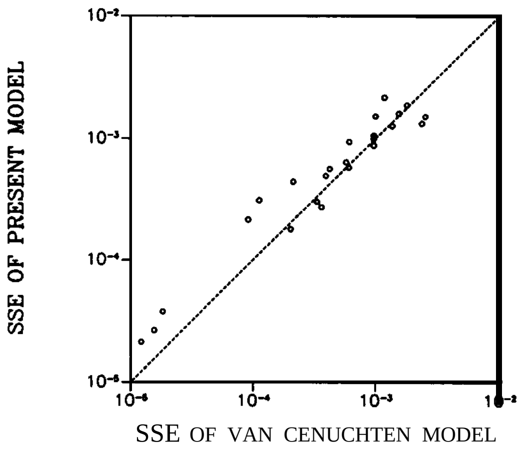

FIG. 8. Comparison of calculated sum of squared errors (SSE) for 25 soils obtained with the proposed model and the equations of van Genuchten (1980).

TABLE 1

Simultaneously fitted parameters *for the* retention model *of Eq.* (6) and *the* conductivity functions given *by Eqs.* (18) through (20)

Soil name	θ_r	θ_s	α (cm^{-1})	c_2	K_s[a]	l	r^2
Silt Loam GE3	0.102	0.403	0.0101	0.270	1.068	0.0	0.997
Sarpy Loam	0.032	0.370	0.250	0.0300	0.985	0.0	0.994
Yolo Light Clay[b]	0.206	0.495	0.0814	0.155	0.993	-2.91	0.998
Silt Columbia	0.176	0.409	0.0037	0.0221	1.059	-0.287	0.994
Caribou Silt Loam (wetting)	0.300	0.445	0.176	0.137	1.001	-2.02	0.999
Caribou Silt Loam (drying)	0.192	0.445	0.0380	0.0849	0.999	0.946	0.999
Pachappa Fine Sandy Loam	0.058	0.344	0.0178	0.585	1.114	0.0	0.892
Rideau Clay Loam (wetting)	0.283	0.447	33.5	0.00682	1.000	0.0	0.998
Touchet Silt Loam	0.039	0.480	0.0026	6570.	0.868	1.74	0.968
Beit Netofa Clay	0.280	0.501	0.103	0.0140	1.052	0.0	0.988
Silt Mont Cenis	0.044	0.402	0.0402	0.718	0.996	-2.32	0.990
Gilat Sandy Loam	0.137	0.416	0.0145	1.905	1.038	-2.13	0.981
Indio Loam	0.000	0.416	2.03	7.87	1.117	22.2	0.988
Guelph Loam A	0.239	0.511	1.72	22.7	0.955	0.236	0.996
Guelph Loam B	0.226	0.504	47.7	1.098	0.953	0.0	0.995
Rubicon Sandy Loam	0.000	0.381	0.00905	0.962	0.989	0.0	0.978
Gilat Loam	0.078	0.433	0.0123	1.636	1.002	0.0	0.994
Pachappa Loam	0.040	0.425	0.00146	6.314	0.939	-0.470	0.988

[a] K_s is the dimensionless saturated hydraulic conductivity, i.e., normalized with respect to the measured value.

*Data from Moore (1939).

CONCLUSIONS

Two relatively simple retention functions (Eqs. (1) and (6)) are proposed. Expressions for the hydraulic conductivity were derived for a sigmoidal retention function given by Eq. (6), and assuming applicability of the pore-size distribution models of Mualem as well as Childs and Collis-George.

With five parameters, the retention model of Eq. (1) can be used to describe retention data with a bimodal shape. This feature may be useful for characterizing dual-porosity type pore systems typical of many undisturbed, macroporous soils. The four-parameter model given by Eq. (6) can be used for S-shaped retention curves. This model allows one to derive closed-form expressions of the unsaturated hydraulic conductivity. In special cases, these conductivity expressions can be approximated by simple polynomial functions. The retention and conductivity functions were used to fit data of more than 20 soils. For most or all examples considered in this study, satisfactory results were obtained in comparison with the Mualem-van Genuchten models. The retention function given by Eq. (1) with $c_1 \neq 1$ cannot be used in conjunction with existing pore-size distribution theories to derive analytical expressions for the hydraulic conductivity function. However, they may be combined with other hydraulic conductivity functions to empirically characterize soil hydraulic properties.

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